Mathematica 11.3 Integration Test Results

Test results for the 494 problems in "1.3.1 Rational functions.m"

Problem 12: Result is not expressed in closed-form.

$$\int \frac{1}{3 a b + 3 b^2 x + 3 b c x^2 + c^2 x^3} \, dx$$

Optimal (type 3, 188 leaves, 7 steps):

$$-\frac{\text{ArcTan}\left[\frac{b^{1/3}+\frac{2 (b+c x)}{(b^2-3 a c)^{1/3}}}{\sqrt{3} b^{1/3}}\right]}{\sqrt{3} b^{2/3} \left(b^2-3 a c\right)^{2/3}}+\frac{\text{Log}\left[b-b^{1/3} \left(b^2-3 a c\right)^{1/3}+c x\right]}{3 b^{2/3} \left(b^2-3 a c\right)^{2/3}}-$$
$$\text{Log}\left[b^{2/3} \left(b^2-3 a c\right)^{2/3}+b^{1/3} c \left(b^2-3 a c\right)^{1/3} \left(\frac{b}{c}+x\right)+c^2 \left(\frac{b}{c}+x\right)^2\right] \Big/ \left(6 b^{2/3} \left(b^2-3 a c\right)^{2/3}\right)$$

Result (type 7, 63 leaves):

$$\frac{1}{3} \text{RootSum}\left[3 a b+3 b^2 \#1+3 b c \#1^2+c^2 \#1^3 \&, \frac{\text{Log}[x-\#1]}{b^2+2 b c \#1+c^2 \#1^2} \&\right]$$

Problem 13: Result is not expressed in closed-form.

$$\int \frac{1}{\left(3 a b + 3 b^2 x + 3 b c x^2 + c^2 x^3\right)^2} \, dx$$

Optimal (type 3, 245 leaves, 8 steps):

$$-\frac{c \left(\frac{b}{c}+x\right)}{3 b \left(b^2-3 a c\right) \left(3 a b+3 b^2 x+3 b c x^2+c^2 x^3\right)}+$$
$$\frac{2 c \,\text{ArcTan}\left[\frac{b^{1/3}+\frac{2 (b+c x)}{(b^2-3 a c)^{1/3}}}{\sqrt{3} b^{1/3}}\right]}{3 \sqrt{3} b^{5/3} \left(b^2-3 a c\right)^{5/3}}-\frac{2 c \,\text{Log}\left[b-b^{1/3} \left(b^2-3 a c\right)^{1/3}+c x\right]}{9 b^{5/3} \left(b^2-3 a c\right)^{5/3}}+$$
$$\left(c \,\text{Log}\left[b^{2/3} \left(b^2-3 a c\right)^{2/3}+b^{1/3} c \left(b^2-3 a c\right)^{1/3} \left(\frac{b}{c}+x\right)+c^2 \left(\frac{b}{c}+x\right)^2\right]\right) \Big/ \left(9 b^{5/3} \left(b^2-3 a c\right)^{5/3}\right)$$

Result (type 7, 112 leaves):

$$-\frac{1}{9 \left(b^3-3 a b c\right)}\left(\frac{3 (b+c x)}{3 a b+x \left(3 b^2+3 b c x+c^2 x^2\right)}+\right.$$
$$\left.2 c \,\text{RootSum}\left[3 a b+3 b^2 \#1+3 b c \#1^2+c^2 \#1^3 \&, \frac{\text{Log}[x-\#1]}{b^2+2 b c \#1+c^2 \#1^2} \&\right]\right)$$

Problem 14: Result is not expressed in closed-form.

$$\int \frac{1}{\left(3\,a\,b+3\,b^2\,x+3\,b\,c\,x^2+c^2\,x^3\right)^3}\,dx$$

Optimal (type 3, 305 leaves, 9 steps):

$$-\frac{c\left(\frac{b}{c}+x\right)}{6\,b\left(b^2-3\,a\,c\right)\left(3\,a\,b+3\,b^2\,x+3\,b\,c\,x^2+c^2\,x^3\right)^2}+\frac{5\,c^2\left(\frac{b}{c}+x\right)}{18\,b^2\left(b^2-3\,a\,c\right)^2\left(3\,a\,b+3\,b^2\,x+3\,b\,c\,x^2+c^2\,x^3\right)}-$$
$$\frac{5\,c^2\,\mathrm{ArcTan}\left[\frac{b^{1/3}+\frac{2\,(b+c\,x)}{\left(b^2-3\,a\,c\right)^{1/3}}}{\sqrt{3}\,b^{1/3}}\right]}{9\,\sqrt{3}\,b^{8/3}\left(b^2-3\,a\,c\right)^{8/3}}+\frac{5\,c^2\,\mathrm{Log}\left[b-b^{1/3}\left(b^2-3\,a\,c\right)^{1/3}+c\,x\right]}{27\,b^{8/3}\left(b^2-3\,a\,c\right)^{8/3}}-$$
$$\left(5\,c^2\,\mathrm{Log}\left[b^{2/3}\left(b^2-3\,a\,c\right)^{2/3}+b^{1/3}\,c\left(b^2-3\,a\,c\right)^{1/3}\left(\frac{b}{c}+x\right)+c^2\left(\frac{b}{c}+x\right)^2\right]\right)\Big/\left(54\,b^{8/3}\left(b^2-3\,a\,c\right)^{8/3}\right)$$

Result (type 7, 149 leaves):

$$\frac{1}{54\left(b^3-3\,a\,b\,c\right)^2}\left(-\frac{3\,(b+c\,x)\left(3\,b^3-15\,b^2\,c\,x-5\,c^3\,x^3-3\,b\,c\left(8\,a+5\,c\,x^2\right)\right)}{\left(3\,a\,b+x\left(3\,b^2+3\,b\,c\,x+c^2\,x^2\right)\right)^2}+\right.$$
$$\left.10\,c^2\,\mathrm{RootSum}\left[3\,a\,b+3\,b^2\,\#1+3\,b\,c\,\#1^2+c^2\,\#1^3\,\&,\ \frac{\mathrm{Log}[x-\#1]}{b^2+2\,b\,c\,\#1+c^2\,\#1^2}\,\&\right]\right)$$

Problem 31: Result more than twice size of optimal antiderivative.

$$\int \left(b\,x+c\,x^2+d\,x^3\right)^n\,dx$$

Optimal (type 6, 132 leaves, 3 steps):

$$\frac{1}{1+n}x\left(1+\frac{2\,d\,x}{c-\sqrt{c^2-4\,b\,d}}\right)^{-n}\left(1+\frac{2\,d\,x}{c+\sqrt{c^2-4\,b\,d}}\right)^{-n}\left(b\,x+c\,x^2+d\,x^3\right)^n$$
$$\mathrm{AppellF1}\left[1+n,\,-n,\,-n,\,2+n,\,-\frac{2\,d\,x}{c-\sqrt{c^2-4\,b\,d}},\,-\frac{2\,d\,x}{c+\sqrt{c^2-4\,b\,d}}\right]$$

Result (type 6, 438 leaves):

$$\left(2^{-1-n}\,d\left(c+\sqrt{c^2-4\,b\,d}\right)(2+n)\,x^2\left(\frac{c-\sqrt{c^2-4\,b\,d}}{2\,d}+x\right)^{-n}\right.$$
$$\left(\frac{c-\sqrt{c^2-4\,b\,d}+2\,d\,x}{d}\right)^{1+n}\left(2\,b+\left(c-\sqrt{c^2-4\,b\,d}\right)x\right)^2\left(x\,(b+x\,(c+d\,x))\right)^{-1+n}$$
$$\left.\text{AppellF1}\left[1+n,-n,-n,2+n,-\frac{2\,d\,x}{c+\sqrt{c^2-4\,b\,d}},\frac{2\,d\,x}{-c+\sqrt{c^2-4\,b\,d}}\right]\right)\Big/$$
$$\left(\left(-c+\sqrt{c^2-4\,b\,d}\right)(1+n)\left(c+\sqrt{c^2-4\,b\,d}+2\,d\,x\right)\right.$$
$$\left(-2\,b\,(2+n)\,\text{AppellF1}\left[1+n,-n,-n,2+n,-\frac{2\,d\,x}{c+\sqrt{c^2-4\,b\,d}},\frac{2\,d\,x}{-c+\sqrt{c^2-4\,b\,d}}\right]+\right.$$
$$n\,x\left(\left(-c+\sqrt{c^2-4\,b\,d}\right)\text{AppellF1}\left[2+n,1-n,-n,3+n,-\frac{2\,d\,x}{c+\sqrt{c^2-4\,b\,d}},\frac{2\,d\,x}{-c+\sqrt{c^2-4\,b\,d}}\right]-\right.$$
$$\left.\left.\left.\left(c+\sqrt{c^2-4\,b\,d}\right)\text{AppellF1}\left[2+n,-n,1-n,3+n,-\frac{2\,d\,x}{c+\sqrt{c^2-4\,b\,d}},\frac{2\,d\,x}{-c+\sqrt{c^2-4\,b\,d}}\right]\right)\right)\right)$$

Problem 32: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\left(a+d\,x^3\right)^n\,dx$$

Optimal (type 5, 35 leaves, 2 steps):

$$\frac{x\left(a+d\,x^3\right)^{1+n}\text{Hypergeometric2F1}\left[1,\frac{4}{3}+n,\frac{4}{3},-\frac{d\,x^3}{a}\right]}{a}$$

Result (type 6, 196 leaves):

$$\frac{1}{d^{1/3}\,(1+n)}2^{-n}\left((-1)^{2/3}\,a^{1/3}+d^{1/3}\,x\right)\left(\frac{a^{1/3}+(-1)^{2/3}\,d^{1/3}\,x}{\left(1+(-1)^{1/3}\right)a^{1/3}}\right)^{-n}\left(\frac{i\left(1+\frac{d^{1/3}\,x}{a^{1/3}}\right)}{3\,i+\sqrt{3}}\right)^{-n}$$
$$\left(a+d\,x^3\right)^n\,\text{AppellF1}\left[1+n,-n,-n,2+n,-\frac{i\left((-1)^{2/3}\,a^{1/3}+d^{1/3}\,x\right)}{\sqrt{3}\,a^{1/3}},\frac{i+\sqrt{3}-\frac{2\,i\,d^{1/3}\,x}{a^{1/3}}}{3\,i+\sqrt{3}}\right]$$

Problem 37: Result is not expressed in closed-form.

$$\int\frac{1}{4\,a\,c+4\,c^2\,x^2+4\,c\,d\,x^3+d^2\,x^4}\,dx$$

Optimal (type 3, 529 leaves, 10 steps):

$$-\frac{d\,\text{ArcTanh}\left[\frac{\sqrt{2}\,c+c^{1/4}\sqrt{c^{3/2}+\sqrt{c^3+4\,a\,d^2}}+\sqrt{2}\,d\,x}{c^{1/4}\sqrt{c^{3/2}-\sqrt{c^3+4\,a\,d^2}}}\right]}{2\sqrt{2}\,c^{3/4}\sqrt{c^3+4\,a\,d^2}\,\sqrt{c^{3/2}-\sqrt{c^3+4\,a\,d^2}}}+\frac{d\,\text{ArcTanh}\left[\frac{c^{1/4}\sqrt{c^{3/2}+\sqrt{c^3+4\,a\,d^2}}-\sqrt{2}\,(c+d\,x)}{c^{1/4}\sqrt{c^{3/2}-\sqrt{c^3+4\,a\,d^2}}}\right]}{2\sqrt{2}\,c^{3/4}\sqrt{c^3+4\,a\,d^2}\,\sqrt{c^{3/2}-\sqrt{c^3+4\,a\,d^2}}}-$$

$$\left(d\,\text{Log}\left[\sqrt{c}\,\sqrt{c^3+4\,a\,d^2}-\sqrt{2}\,c^{1/4}\,d\,\sqrt{c^{3/2}+\sqrt{c^3+4\,a\,d^2}}\,\left(\frac{c}{d}+x\right)+d^2\left(\frac{c}{d}+x\right)^2\right]\right)\Big/$$

$$\left(4\sqrt{2}\,c^{3/4}\sqrt{c^3+4\,a\,d^2}\,\sqrt{c^{3/2}+\sqrt{c^3+4\,a\,d^2}}\right)+$$

$$\left(d\,\text{Log}\left[\sqrt{c}\,\sqrt{c^3+4\,a\,d^2}+\sqrt{2}\,c^{1/4}\,d\,\sqrt{c^{3/2}+\sqrt{c^3+4\,a\,d^2}}\,\left(\frac{c}{d}+x\right)+d^2\left(\frac{c}{d}+x\right)^2\right]\right)\Big/$$

$$\left(4\sqrt{2}\,c^{3/4}\sqrt{c^3+4\,a\,d^2}\,\sqrt{c^{3/2}+\sqrt{c^3+4\,a\,d^2}}\right)$$

Result (type 7, 71 leaves) :

$$\frac{1}{4}\,\text{RootSum}\left[4\,a\,c+4\,c^2\,\#1^2+4\,c\,d\,\#1^3+d^2\,\#1^4\,\&,\ \frac{\text{Log}[x-\#1]}{2\,c^2\,\#1+3\,c\,d\,\#1^2+d^2\,\#1^3}\,\&\right]$$

Problem 38: Result is not expressed in closed-form.

$$\int\frac{1}{\left(4\,a\,c+4\,c^2\,x^2+4\,c\,d\,x^3+d^2\,x^4\right)^2}\,dx$$

Optimal (type 3, 746 leaves, 11 steps) :

$$-\frac{\left(\frac{c}{d}+x\right)\left(c^3-4ad^2-cd^2\left(\frac{c}{d}+x\right)^2\right)}{16ac\left(c^3+4ad^2\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)}-$$

$$\left(d\left(c^3+12ad^2+c^{3/2}\sqrt{c^3+4ad^2}\right)\text{ArcTanh}\left[\frac{\sqrt{2}\,c+c^{1/4}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}+\sqrt{2}\,dx}{c^{1/4}\sqrt{c^{3/2}-\sqrt{c^3+4ad^2}}}\right]\right)\Big/$$

$$\left(32\sqrt{2}\,ac^{7/4}\left(c^3+4ad^2\right)^{3/2}\sqrt{c^{3/2}-\sqrt{c^3+4ad^2}}\right)+$$

$$\left(d\left(c^3+12ad^2+c^{3/2}\sqrt{c^3+4ad^2}\right)\text{ArcTanh}\left[\frac{c^{1/4}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}-\sqrt{2}\,(c+dx)}{c^{1/4}\sqrt{c^{3/2}-\sqrt{c^3+4ad^2}}}\right]\right)\Big/$$

$$\left(32\sqrt{2}\,ac^{7/4}\left(c^3+4ad^2\right)^{3/2}\sqrt{c^{3/2}-\sqrt{c^3+4ad^2}}\right)-\left(d\left(c^3+12ad^2-c^{3/2}\sqrt{c^3+4ad^2}\right)\right.$$

$$\left.\text{Log}\left[\sqrt{c}\sqrt{c^3+4ad^2}-\sqrt{2}\,c^{1/4}d\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)+d^2\left(\frac{c}{d}+x\right)^2\right]\right)\Big/$$

$$\left(64\sqrt{2}\,ac^{7/4}\left(c^3+4ad^2\right)^{3/2}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\right)+\left(d\left(c^3+12ad^2-c^{3/2}\sqrt{c^3+4ad^2}\right)\right.$$

$$\left.\text{Log}\left[\sqrt{c}\sqrt{c^3+4ad^2}+\sqrt{2}\,c^{1/4}d\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)+d^2\left(\frac{c}{d}+x\right)^2\right]\right)\Big/$$

$$\left(64\sqrt{2}\,ac^{7/4}\left(c^3+4ad^2\right)^{3/2}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\right)$$

Result (type 7, 182 leaves):

$$\left(\frac{4(c+dx)\left(4ad+cx(2c+dx)\right)}{4ac+x^2(2c+dx)^2}+\text{RootSum}\left[4ac+4c^2\#1^2+4cd\#1^3+d^2\#1^4\,\&,\right.\right.$$

$$\left(2c^3\,\text{Log}[x-\#1]+12ad^2\,\text{Log}[x-\#1]+2c^2d\,\text{Log}[x-\#1]\,\#1+cd^2\,\text{Log}[x-\#1]\,\#1^2\right)\Big/$$

$$\left.\left.\left(2c^2\#1+3cd\#1^2+d^2\#1^3\right)\&\right]\right)\Big/\left(64ac\left(c^3+4ad^2\right)\right)$$

Problem 43: Result is not expressed in closed-form.

$$\int\frac{1}{8ae^2-d^3x+8de^2x^3+8e^3x^4}\,dx$$

Optimal (type 3, 153 leaves, 4 steps):

$$\frac{2\,\text{ArcTanh}\left[\frac{d+4\,e\,x}{\sqrt{3\,d^2-2\,\sqrt{d^4-64\,a\,e^3}}}\right]}{\sqrt{d^4-64\,a\,e^3}\,\sqrt{3\,d^2-2\,\sqrt{d^4-64\,a\,e^3}}}-\frac{2\,\text{ArcTanh}\left[\frac{d+4\,e\,x}{\sqrt{3\,d^2+2\,\sqrt{d^4-64\,a\,e^3}}}\right]}{\sqrt{d^4-64\,a\,e^3}\,\sqrt{3\,d^2+2\,\sqrt{d^4-64\,a\,e^3}}}$$

Result (type 7, 71 leaves):

$$-\text{RootSum}\left[8\,a\,e^2-d^3\,\#1+8\,d\,e^2\,\#1^3+8\,e^3\,\#1^4\,\&,\,\frac{\text{Log}[x-\#1]}{d^3-24\,d\,e^2\,\#1^2-32\,e^3\,\#1^3}\,\&\right]$$

Problem 44: Result is not expressed in closed-form.

$$\int\frac{1}{\left(8\,a\,e^2-d^3\,x+8\,d\,e^2\,x^3+8\,e^3\,x^4\right)^2}\,dx$$

Optimal (type 3, 342 leaves, 5 steps):

$$\frac{2\,e\left(\frac{d}{4\,e}+x\right)\left(13\,d^4-256\,a\,e^3-48\,d^2\,e^2\left(\frac{d}{4\,e}+x\right)^2\right)}{\left(5\,d^8-64\,a\,d^4\,e^3-16\,384\,a^2\,e^6\right)\left(8\,a\,e^2-d^3\,x+8\,d\,e^2\,x^3+8\,e^3\,x^4\right)}-\frac{24\,e\left(d^4+128\,a\,e^3-d^2\,\sqrt{d^4-64\,a\,e^3}\right)\text{ArcTanh}\left[\frac{d+4\,e\,x}{\sqrt{3\,d^2-2\,\sqrt{d^4-64\,a\,e^3}}}\right]}{\left(d^4-64\,a\,e^3\right)^{3/2}\left(5\,d^4+256\,a\,e^3\right)\sqrt{3\,d^2-2\,\sqrt{d^4-64\,a\,e^3}}}+\frac{24\,e\left(d^4+128\,a\,e^3+d^2\,\sqrt{d^4-64\,a\,e^3}\right)\text{ArcTanh}\left[\frac{d+4\,e\,x}{\sqrt{3\,d^2+2\,\sqrt{d^4-64\,a\,e^3}}}\right]}{\left(d^4-64\,a\,e^3\right)^{3/2}\left(5\,d^4+256\,a\,e^3\right)\sqrt{3\,d^2+2\,\sqrt{d^4-64\,a\,e^3}}}$$

Result (type 7, 234 leaves):

$$\frac{(d+4\,e\,x)\left(5\,d^4-128\,a\,e^3-12\,d^3\,e\,x-24\,d^2\,e^2\,x^2\right)}{\left(d^4-64\,a\,e^3\right)\left(5\,d^4+256\,a\,e^3\right)\left(8\,a\,e^2-d^3\,x+8\,d\,e^2\,x^3+8\,e^3\,x^4\right)}+\left(48\,e^2\,\text{RootSum}\left[8\,a\,e^2-d^3\,\#1+8\,d\,e^2\,\#1^3+8\,e^3\,\#1^4\,\&,\,\frac{32\,a\,e^2\,\text{Log}[x-\#1]+d^3\,\text{Log}[x-\#1]\,\#1+2\,d^2\,e\,\text{Log}[x-\#1]\,\#1^2}{-d^3+24\,d\,e^2\,\#1^2+32\,e^3\,\#1^3}\,\&\right]\right)\Big/\left(-5\,d^8+64\,a\,d^4\,e^3+16\,384\,a^2\,e^6\right)$$

Problem 49: Result is not expressed in closed-form.

$$\int\frac{1}{8+8\,x-x^3+8\,x^4}\,dx$$

Optimal (type 3, 268 leaves, 16 steps):

$$-\frac{\text{ArcTan}\left[\frac{3-\left(1+\frac{4}{x}\right)^2}{6\sqrt{7}}\right]}{12\sqrt{7}}-\frac{1}{12}\sqrt{\frac{109+67\sqrt{29}}{1218}}\ \text{ArcTan}\left[\frac{2-\sqrt{6\left(1+\sqrt{29}\right)}+\frac{8}{x}}{\sqrt{6\left(-1+\sqrt{29}\right)}}\right]-$$

$$\frac{1}{12}\sqrt{\frac{109+67\sqrt{29}}{1218}}\ \text{ArcTan}\left[\frac{2+\sqrt{6\left(1+\sqrt{29}\right)}+\frac{8}{x}}{\sqrt{6\left(-1+\sqrt{29}\right)}}\right]-$$

$$\frac{1}{24}\sqrt{\frac{-109+67\sqrt{29}}{1218}}\ \text{Log}\left[3\sqrt{29}-\sqrt{6\left(1+\sqrt{29}\right)}\left(1+\frac{4}{x}\right)+\left(1+\frac{4}{x}\right)^2\right]+$$

$$\frac{1}{24}\sqrt{\frac{-109+67\sqrt{29}}{1218}}\ \text{Log}\left[3\sqrt{29}+\sqrt{6\left(1+\sqrt{29}\right)}\left(1+\frac{4}{x}\right)+\left(1+\frac{4}{x}\right)^2\right]$$

Result (type 7, 45 leaves):

$$\text{RootSum}\left[8+8\,\#1-\#1^3+8\,\#1^4\ \&,\ \frac{\text{Log}[x-\#1]}{8-3\,\#1^2+32\,\#1^3}\ \&\right]$$

Problem 50: Result is not expressed in closed-form.

$$\int\frac{1}{\left(8+8x-x^3+8x^4\right)^2}\,dx$$

Optimal (type 3, 357 leaves, 18 steps):

$$-\frac{207+29\left(1+\frac{4}{x}\right)^2}{336\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)}+\frac{5\left(5157+199\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)}{87696\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)}-$$

$$\frac{17\,\text{ArcTan}\left[\frac{3-\left(1+\frac{4}{x}\right)^2}{6\sqrt{7}}\right]}{1008\sqrt{7}}-\frac{\sqrt{\frac{180983329+45923327\sqrt{29}}{1218}}\,\text{ArcTan}\left[\frac{2-\sqrt{6\left(1+\sqrt{29}\right)}+\frac{8}{x}}{\sqrt{6\left(-1+\sqrt{29}\right)}}\right]}{87696}-$$

$$\frac{\sqrt{\frac{180983329+45923327\sqrt{29}}{1218}}\,\text{ArcTan}\left[\frac{2+\sqrt{6\left(1+\sqrt{29}\right)}+\frac{8}{x}}{\sqrt{6\left(-1+\sqrt{29}\right)}}\right]}{87696}-\frac{1}{175392}$$

$$\sqrt{\frac{-180983329+45923327\sqrt{29}}{1218}}\,\text{Log}\left[3\sqrt{29}-\sqrt{6\left(1+\sqrt{29}\right)}\left(1+\frac{4}{x}\right)+\left(1+\frac{4}{x}\right)^2\right]+$$

$$\frac{1}{175392}\sqrt{\frac{-180983329+45923327\sqrt{29}}{1218}}\,\text{Log}\left[3\sqrt{29}+\sqrt{6\left(1+\sqrt{29}\right)}\left(1+\frac{4}{x}\right)+\left(1+\frac{4}{x}\right)^2\right]$$

Result (type 7, 113 leaves):

$$\frac{544+1539\,x-1146\,x^2+784\,x^3}{43848\left(8+8\,x-x^3+8\,x^4\right)}+\frac{1}{21924}$$

$$\text{RootSum}\left[8+8\,\#1-\#1^3+8\,\#1^4\,\&,\ \frac{2243\,\text{Log}[x-\#1]-1097\,\text{Log}[x-\#1]\,\#1+392\,\text{Log}[x-\#1]\,\#1^2}{8-3\,\#1^2+32\,\#1^3}\,\&\right]$$

Problem 55: Result is not expressed in closed-form.

$$\int\frac{1}{1+4\,x+4\,x^2+4\,x^4}\,dx$$

Optimal (type 3, 234 leaves, 15 steps):

$$\frac{1}{2}\,\text{ArcTan}\left[\frac{1}{2}\left(-1+\left(1+\frac{1}{x}\right)^2\right)\right]-\frac{1}{2}\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\,\text{ArcTan}\left[\frac{2-\sqrt{2\left(1+\sqrt{5}\right)}+\frac{2}{x}}{\sqrt{2\left(-1+\sqrt{5}\right)}}\right]-$$

$$\frac{1}{2}\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\,\text{ArcTan}\left[\frac{2+\sqrt{2\left(1+\sqrt{5}\right)}+\frac{2}{x}}{\sqrt{2\left(-1+\sqrt{5}\right)}}\right]-$$

$$\frac{1}{4}\sqrt{\frac{1}{5}\left(-2+\sqrt{5}\right)}\,\text{Log}\left[\sqrt{5}-\sqrt{2\left(1+\sqrt{5}\right)}\,\left(1+\frac{1}{x}\right)+\left(1+\frac{1}{x}\right)^2\right]+$$

$$\frac{1}{4}\sqrt{\frac{1}{5}\left(-2+\sqrt{5}\right)}\,\text{Log}\left[\sqrt{5}+\sqrt{2\left(1+\sqrt{5}\right)}\,\left(1+\frac{1}{x}\right)+\left(1+\frac{1}{x}\right)^2\right]$$

Result (type 7, 47 leaves):

$$\frac{1}{4}\,\text{RootSum}\left[1+4\,\sharp1+4\,\sharp1^2+4\,\sharp1^4\,\&,\,\frac{\text{Log}[x-\sharp1]}{1+2\,\sharp1+4\,\sharp1^3}\,\&\right]$$

Problem 56: Result is not expressed in closed-form.

$$\int\frac{1}{\left(1+4\,x+4\,x^2+4\,x^4\right)^2}\,dx$$

Optimal (type 3, 317 leaves, 17 steps):

$$-\frac{17-\left(1+\frac{1}{x}\right)^2}{2\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)}+\frac{\left(59-17\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)}{10\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)}+$$

$$\frac{7}{4}\,\text{ArcTan}\left[\frac{1}{2}\left(-1+\left(1+\frac{1}{x}\right)^2\right)\right]-\frac{1}{20}\sqrt{\frac{1}{10}\left(5959+2665\sqrt{5}\right)}\;\text{ArcTan}\left[\frac{2-\sqrt{2\left(1+\sqrt{5}\right)}+\frac{2}{x}}{\sqrt{2\left(-1+\sqrt{5}\right)}}\right]-$$

$$\frac{1}{20}\sqrt{\frac{1}{10}\left(5959+2665\sqrt{5}\right)}\;\text{ArcTan}\left[\frac{2+\sqrt{2\left(1+\sqrt{5}\right)}+\frac{2}{x}}{\sqrt{2\left(-1+\sqrt{5}\right)}}\right]+$$

$$\frac{1}{40}\sqrt{\frac{1}{10}\left(-5959+2665\sqrt{5}\right)}\;\text{Log}\left[\sqrt{5}-\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\left(1+\frac{1}{x}\right)^2\right]-$$

$$\frac{1}{40}\sqrt{\frac{1}{10}\left(-5959+2665\sqrt{5}\right)}\;\text{Log}\left[\sqrt{5}+\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\left(1+\frac{1}{x}\right)^2\right]$$

Result (type 7, 108 leaves):

$$\frac{1}{40}\left(\frac{38+84x-32x^2+72x^3}{1+4x+4x^2+4x^4}+\text{RootSum}\left[1+4\#1+4\#1^2+4\#1^4\,\&,\,\frac{27\,\text{Log}[x-\#1]-16\,\text{Log}[x-\#1]\,\#1+18\,\text{Log}[x-\#1]\,\#1^2}{1+2\#1+4\#1^3}\,\&\right]\right)$$

Problem 61: Result is not expressed in closed-form.

$$\int\frac{1}{8+24x+8x^2-15x^3+8x^4}\,dx$$

Optimal (type 3, 263 leaves, 16 steps):

$$-\frac{1}{4}\sqrt{\frac{5167+235\sqrt{517}}{40326}}\ \mathrm{ArcTan}\left[\frac{6-\sqrt{2\left(19+\sqrt{517}\right)}+\frac{8}{x}}{\sqrt{2\left(-19+\sqrt{517}\right)}}\right]-$$

$$\frac{1}{4}\sqrt{\frac{5167+235\sqrt{517}}{40326}}\ \mathrm{ArcTan}\left[\frac{6+\sqrt{2\left(19+\sqrt{517}\right)}+\frac{8}{x}}{\sqrt{2\left(-19+\sqrt{517}\right)}}\right]+\frac{1}{4}\sqrt{\frac{3}{13}}\ \mathrm{ArcTan}\left[\frac{8+12x-5x^2}{\sqrt{39}\ x^2}\right]-$$

$$\frac{1}{8}\sqrt{\frac{-5167+235\sqrt{517}}{40326}}\ \mathrm{Log}\left[\sqrt{517}-\sqrt{2\left(19+\sqrt{517}\right)}\left(3+\frac{4}{x}\right)+\left(3+\frac{4}{x}\right)^2\right]+$$

$$\frac{1}{8}\sqrt{\frac{-5167+235\sqrt{517}}{40326}}\ \mathrm{Log}\left[\sqrt{517}+\sqrt{2\left(19+\sqrt{517}\right)}\left(3+\frac{4}{x}\right)+\left(3+\frac{4}{x}\right)^2\right]$$

Result (type 7, 55 leaves):

$$\mathrm{RootSum}\left[8+24\,\#1+8\,\#1^2-15\,\#1^3+8\,\#1^4\ \&,\ \frac{\mathrm{Log}[x-\#1]}{24+16\,\#1-45\,\#1^2+32\,\#1^3}\ \&\right]$$

Problem 62: Result is not expressed in closed-form.

$$\int\frac{1}{\left(8+24x+8x^2-15x^3+8x^4\right)^2}\,dx$$

Optimal (type 3, 366 leaves, 18 steps):

$$-\frac{3\left(3359-107\left(3+\frac{4}{x}\right)^2\right)}{208\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)}+\frac{\left(3327931-129631\left(3+\frac{4}{x}\right)^2\right)\left(3+\frac{4}{x}\right)}{322608\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)}-$$

$$\frac{\sqrt{\frac{19+\sqrt{517}}{40326}}\left(1678181+74897\sqrt{517}\right)\text{ArcTan}\left[\frac{6-\sqrt{2\left(19+\sqrt{517}\right)}+\frac{8}{x}}{\sqrt{2\left(-19+\sqrt{517}\right)}}\right]}{645216}-$$

$$\frac{\sqrt{\frac{19+\sqrt{517}}{40326}}\left(1678181+74897\sqrt{517}\right)\text{ArcTan}\left[\frac{6+\sqrt{2\left(19+\sqrt{517}\right)}+\frac{8}{x}}{\sqrt{2\left(-19+\sqrt{517}\right)}}\right]}{645216}+$$

$$\frac{73}{208}\sqrt{\frac{3}{13}}\text{ArcTan}\left[\frac{8+12x-5x^2}{\sqrt{39}\,x^2}\right]-\frac{1}{645216}\sqrt{\frac{-59644114671451+2623170438295\sqrt{517}}{40326}}$$

$$\text{Log}\left[\sqrt{517}-\sqrt{2\left(19+\sqrt{517}\right)}\left(3+\frac{4}{x}\right)+\left(3+\frac{4}{x}\right)^2\right]+\frac{1}{645216}$$

$$\sqrt{\frac{-59644114671451+2623170438295\sqrt{517}}{40326}}\text{Log}\left[\sqrt{517}+\sqrt{2\left(19+\sqrt{517}\right)}\left(3+\frac{4}{x}\right)+\left(3+\frac{4}{x}\right)^2\right]$$

Result (type 7, 128 leaves):

$$\frac{72888+89033x-94314x^2+39280x^3}{161304\left(8+24x+8x^2-15x^3+8x^4\right)}+\frac{1}{80652}\text{RootSum}\Big[8+24\,\#1+8\,\#1^2-15\,\#1^3+8\,\#1^4\,\&,$$

$$\frac{74897\,\text{Log}[x-\#1]-57489\,\text{Log}[x-\#1]\,\#1+19640\,\text{Log}[x-\#1]\,\#1^2}{24+16\,\#1-45\,\#1^2+32\,\#1^3}\,\&\Big]$$

Problem 65: Result more than twice size of optimal antiderivative.

$$\int\left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)dx$$

Optimal (type 1, 14 leaves, 1 step):

$$\frac{(a+bx)^6}{6b}$$

Result (type 1, 61 leaves):

$$a^5x+\frac{5}{2}a^4bx^2+\frac{10}{3}a^3b^2x^3+\frac{5}{2}a^2b^3x^4+ab^4x^5+\frac{b^5x^6}{6}$$

Problem 92: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{1-(c+dx)^2}\,dx$$

Optimal (type 3, 10 leaves, 2 steps):

$$\frac{\text{ArcTanh}[c+d\,x]}{d}$$

Result (type 3, 32 leaves):

$$-\frac{\text{Log}[1-c-d\,x]}{2\,d}+\frac{\text{Log}[1+c+d\,x]}{2\,d}$$

Problem 95: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{1-(1+x)^2}\,dx$$

Optimal (type 3, 4 leaves, 2 steps):

$$\text{ArcTanh}[1+x]$$

Result (type 3, 15 leaves):

$$-\frac{\text{Log}[x]}{2}+\frac{1}{2}\,\text{Log}[2+x]$$

Problem 103: Result is not expressed in closed-form.

$$\int \frac{x^3}{a+b\,(c+d\,x)^3}\,dx$$

Optimal (type 3, 234 leaves, 11 steps):

$$\frac{x}{b\,d^3}+\frac{\left(a-3\,a^{1/3}\,b^{2/3}\,c^2+b\,c^3\right)\,\text{ArcTan}\left[\frac{a^{1/3}-2\,b^{1/3}\,(c+d\,x)}{\sqrt{3}\,a^{1/3}}\right]}{\sqrt{3}\,a^{2/3}\,b^{4/3}\,d^4}-\frac{\left(a+3\,a^{1/3}\,b^{2/3}\,c^2+b\,c^3\right)\,\text{Log}\left[a^{1/3}+b^{1/3}\,(c+d\,x)\right]}{3\,a^{2/3}\,b^{4/3}\,d^4}+\frac{\left(a+3\,a^{1/3}\,b^{2/3}\,c^2+b\,c^3\right)\,\text{Log}\left[a^{2/3}-a^{1/3}\,b^{1/3}\,(c+d\,x)+b^{2/3}\,(c+d\,x)^2\right]}{6\,a^{2/3}\,b^{4/3}\,d^4}-\frac{c\,\text{Log}\left[a+b\,(c+d\,x)^3\right]}{b\,d^4}$$

Result (type 7, 132 leaves):

$$-\frac{1}{3\,b^2\,d^4}\left(-3\,b\,d\,x+\text{RootSum}\left[a+b\,c^3+3\,b\,c^2\,d\,\#1+3\,b\,c\,d^2\,\#1^2+b\,d^3\,\#1^3\,\&,\ \left(a\,\text{Log}[x-\#1]+b\,c^3\,\text{Log}[x-\#1]+3\,b\,c^2\,d\,\text{Log}[x-\#1]\,\#1+3\,b\,c\,d^2\,\text{Log}[x-\#1]\,\#1^2\right)\Big/\left(c^2+2\,c\,d\,\#1+d^2\,\#1^2\right)\,\&\right]\right)$$

Problem 104: Result is not expressed in closed-form.

$$\int \frac{x^2}{a+b\,(c+d\,x)^3}\,dx$$

Optimal (type 3, 210 leaves, 9 steps):

$$\frac{c\left(2a^{1/3}-b^{1/3}c\right)\text{ArcTan}\left[\frac{a^{1/3}-2b^{1/3}(c+d x)}{\sqrt{3}\,a^{1/3}}\right]}{\sqrt{3}\,a^{2/3}b^{2/3}d^3}+\frac{c\left(2a^{1/3}+b^{1/3}c\right)\text{Log}\left[a^{1/3}+b^{1/3}(c+d x)\right]}{3a^{2/3}b^{2/3}d^3}-\frac{c\left(2a^{1/3}+b^{1/3}c\right)\text{Log}\left[a^{2/3}-a^{1/3}b^{1/3}(c+d x)+b^{2/3}(c+d x)^2\right]}{6a^{2/3}b^{2/3}d^3}+\frac{\text{Log}\left[a+b(c+d x)^3\right]}{3b d^3}$$

Result (type 7, 81 leaves):

$$\frac{1}{3 b d}\text{RootSum}\left[a+b c^3+3 b c^2 d \#1+3 b c d^2 \#1^2+b d^3 \#1^3\,\&,\ \frac{\text{Log}[x-\#1]\,\#1^2}{c^2+2 c d \#1+d^2 \#1^2}\,\&\right]$$

Problem 105: Result is not expressed in closed-form.

$$\int \frac{x}{a+b\,(c+d x)^3}\,dx$$

Optimal (type 3, 180 leaves, 7 steps):

$$-\frac{\left(a^{1/3}-b^{1/3}c\right)\text{ArcTan}\left[\frac{a^{1/3}-2b^{1/3}(c+d x)}{\sqrt{3}\,a^{1/3}}\right]}{\sqrt{3}\,a^{2/3}b^{2/3}d^2}-\frac{\left(a^{1/3}+b^{1/3}c\right)\text{Log}\left[a^{1/3}+b^{1/3}(c+d x)\right]}{3a^{2/3}b^{2/3}d^2}+\frac{\left(a^{1/3}+b^{1/3}c\right)\text{Log}\left[a^{2/3}-a^{1/3}b^{1/3}(c+d x)+b^{2/3}(c+d x)^2\right]}{6a^{2/3}b^{2/3}d^2}$$

Result (type 7, 79 leaves):

$$\frac{1}{3 b d}\text{RootSum}\left[a+b c^3+3 b c^2 d \#1+3 b c d^2 \#1^2+b d^3 \#1^3\,\&,\ \frac{\text{Log}[x-\#1]\,\#1}{c^2+2 c d \#1+d^2 \#1^2}\,\&\right]$$

Problem 107: Result is not expressed in closed-form.

$$\int \frac{1}{x\left(a+b\,(c+d x)^3\right)}\,dx$$

Optimal (type 3, 224 leaves, 11 steps):

$$\frac{b^{1/3}c\,\text{ArcTan}\left[\frac{a^{1/3}-2b^{1/3}(c+d x)}{\sqrt{3}\,a^{1/3}}\right]}{\sqrt{3}\,a^{2/3}\left(a^{2/3}-a^{1/3}b^{1/3}c+b^{2/3}c^2\right)}+\frac{\text{Log}[x]}{a+b c^3}-\frac{\text{Log}\left[a^{1/3}+b^{1/3}(c+d x)\right]}{3a^{2/3}\left(a^{1/3}+b^{1/3}c\right)}-\frac{\left(2a^{1/3}-b^{1/3}c\right)\text{Log}\left[a^{2/3}-a^{1/3}b^{1/3}(c+d x)+b^{2/3}(c+d x)^2\right]}{6a^{2/3}\left(a^{2/3}-a^{1/3}b^{1/3}c+b^{2/3}c^2\right)}$$

Result (type 7, 119 leaves):

$$-\frac{1}{3\left(a+b c^3\right)}\left(-3\,\text{Log}[x]+\text{RootSum}\left[a+b c^3+3 b c^2 d \#1+3 b c d^2 \#1^2+b d^3 \#1^3\,\&,\ \frac{3 c^2\,\text{Log}[x-\#1]+3 c d\,\text{Log}[x-\#1]\,\#1+d^2\,\text{Log}[x-\#1]\,\#1^2}{c^2+2 c d \#1+d^2 \#1^2}\,\&\right]\right)$$

Problem 108: Result is not expressed in closed-form.

$$\int \frac{1}{x^2 \left(a + b (c + d x)^3\right)} \, dx$$

Optimal (type 3, 314 leaves, 11 steps):

$$-\frac{1}{\left(a + b c^3\right) x} + \frac{b^{1/3} \left(a^{1/3} - b^{1/3} c\right) \left(a^{1/3} + b^{1/3} c\right)^3 d \, \text{ArcTan}\left[\frac{a^{1/3} - 2 b^{1/3} (c + d x)}{\sqrt{3} \, a^{1/3}}\right]}{\sqrt{3} \, a^{2/3} \left(a + b c^3\right)^2} - \frac{3 b c^2 d \, \text{Log}[x]}{\left(a + b c^3\right)^2} + \frac{b^{1/3} \left(a^{1/3} \left(a - 2 b c^3\right) - b^{1/3} c \left(2 a - b c^3\right)\right) d \, \text{Log}\left[a^{1/3} + b^{1/3} (c + d x)\right]}{3 a^{2/3} \left(a + b c^3\right)^2} - \frac{1}{6 a^{2/3} \left(a + b c^3\right)^2} b^{1/3} \left(a^{1/3} \left(a - 2 b c^3\right) - b^{1/3} c \left(2 a - b c^3\right)\right) d \, \text{Log}\left[a^{2/3} - a^{1/3} b^{1/3} (c + d x) + b^{2/3} (c + d x)^2\right] + \frac{b c^2 d \, \text{Log}\left[a + b (c + d x)^3\right]}{\left(a + b c^3\right)^2}$$

Result (type 7, 173 leaves):

$$\frac{1}{3 \left(a + b c^3\right)^2 x} \left(-3 \left(a + b c^3 + 3 b c^2 d x \, \text{Log}[x]\right) + d x \, \text{RootSum}\left[a + b c^3 + 3 b c^2 d \, \#1 + 3 b c d^2 \, \#1^2 + b d^3 \, \#1^3 \, \&, \left(-3 a c \, \text{Log}[x - \#1] + 6 b c^4 \, \text{Log}[x - \#1] - a d \, \text{Log}[x - \#1] \, \#1 + 8 b c^3 d \, \text{Log}[x - \#1] \, \#1 + 3 b c^2 d^2 \, \text{Log}[x - \#1] \, \#1^2\right) \Big/ \left(c^2 + 2 c d \, \#1 + d^2 \, \#1^2\right) \, \&\right]\right)$$

Problem 109: Result is not expressed in closed-form.

$$\int \frac{1}{x^3 \left(a + b (c + d x)^3\right)} \, dx$$

Optimal (type 3, 393 leaves, 11 steps):

$$-\frac{1}{2 \left(a + b c^3\right) x^2} + \frac{3 b c^2 d}{\left(a + b c^3\right)^2 x} + \frac{b^{2/3} \left(a^{1/3} + b^{1/3} c\right)^3 \left(a - 3 a^{2/3} b^{1/3} c + b c^3\right) d^2 \, \text{ArcTan}\left[\frac{a^{1/3} - 2 b^{1/3} (c + d x)}{\sqrt{3} \, a^{1/3}}\right]}{\sqrt{3} \, a^{2/3} \left(a + b c^3\right)^3} - \frac{3 b c \left(a - 2 b c^3\right) d^2 \, \text{Log}[x]}{\left(a + b c^3\right)^3} - \frac{1}{3 a^{2/3} \left(a + b c^3\right)^3} b^{2/3} \left(a^2 + 6 a^{4/3} b^{2/3} c^2 - 7 a b c^3 - 3 a^{1/3} b^{5/3} c^5 + b^2 c^6\right) d^2 \, \text{Log}\left[a^{1/3} + b^{1/3} (c + d x)\right] + \frac{1}{6 a^{2/3} \left(a + b c^3\right)^3} b^{2/3} \left(a^2 + 6 a^{4/3} b^{2/3} c^2 - 7 a b c^3 - 3 a^{1/3} b^{5/3} c^5 + b^2 c^6\right) d^2 \, \text{Log}\left[a^{2/3} - a^{1/3} b^{1/3} (c + d x) + b^{2/3} (c + d x)^2\right] + \frac{b c \left(a - 2 b c^3\right) d^2 \, \text{Log}\left[a + b (c + d x)^3\right]}{\left(a + b c^3\right)^3}$$

Result (type 7, 244 leaves):

$$-\frac{1}{6\left(a+b c^3\right)^3 x^2}\left(3\left(a+b c^3\right)\left(a+b c^2(c-6 d x)\right)+18 b c\left(a-2 b c^3\right) d^2 x^2 \text{Log}[x]+2 d^2 x^2 \text{RootSum}\left[a+b c^3+3 b c^2 d \#1+3 b c d^2 \#1^2+b d^3 \#1^3 \&,\left(a^2 \text{Log}[x-\#1]-16 a b c^3 \text{Log}[x-\#1]+10 b^2 c^6 \text{Log}[x-\#1]-12 a b c^2 d \text{Log}[x-\#1] \#1+15 b^2 c^5 d \text{Log}[x-\#1] \#1-3 a b c d^2 \text{Log}[x-\#1] \#1^2+6 b^2 c^4 d^2 \text{Log}[x-\#1] \#1^2\right) \Big/ \left(c^2+2 c d \#1+d^2 \#1^2\right) \&\right]\right)$$

Problem 110: Result is not expressed in closed-form.

$$\int \frac{x^3}{a+b (c+d x)^4} \, dx$$

Optimal (type 3, 356 leaves, 16 steps):

$$\frac{3 c^2 \text{ArcTan}\left[\frac{\sqrt{b} (c+d x)^2}{\sqrt{a}}\right]}{2 \sqrt{a} \sqrt{b} d^4}+\frac{c\left(3 \sqrt{a}+\sqrt{b} c^2\right) \text{ArcTan}\left[1-\frac{\sqrt{2} b^{1/4} (c+d x)}{a^{1/4}}\right]}{2 \sqrt{2} a^{3/4} b^{3/4} d^4}-\frac{c\left(3 \sqrt{a}+\sqrt{b} c^2\right) \text{ArcTan}\left[1+\frac{\sqrt{2} b^{1/4} (c+d x)}{a^{1/4}}\right]}{2 \sqrt{2} a^{3/4} b^{3/4} d^4}-\frac{c\left(3 \sqrt{a}-\sqrt{b} c^2\right) \text{Log}\left[\sqrt{a}-\sqrt{2} a^{1/4} b^{1/4} (c+d x)+\sqrt{b} (c+d x)^2\right]}{4 \sqrt{2} a^{3/4} b^{3/4} d^4}+\frac{c\left(3 \sqrt{a}-\sqrt{b} c^2\right) \text{Log}\left[\sqrt{a}+\sqrt{2} a^{1/4} b^{1/4} (c+d x)+\sqrt{b} (c+d x)^2\right]}{4 \sqrt{2} a^{3/4} b^{3/4} d^4}+\frac{\text{Log}\left[a+b (c+d x)^4\right]}{4 b d^4}$$

Result (type 7, 106 leaves):

$$\frac{1}{4 b d} \text{RootSum}\left[a+b c^4+4 b c^3 d \#1+6 b c^2 d^2 \#1^2+4 b c d^3 \#1^3+b d^4 \#1^4 \&,\frac{\text{Log}[x-\#1] \#1^3}{c^3+3 c^2 d \#1+3 c d^2 \#1^2+d^3 \#1^3} \&\right]$$

Problem 111: Result is not expressed in closed-form.

$$\int \frac{x^2}{a+b (c+d x)^4} \, dx$$

Optimal (type 3, 318 leaves, 14 steps):

$$-\frac{c\,\text{ArcTan}\left[\frac{\sqrt{b}\,(c+d\,x)^2}{\sqrt{a}}\right]}{\sqrt{a}\,\sqrt{b}\,d^3}-\frac{\left(\sqrt{a}+\sqrt{b}\,c^2\right)\text{ArcTan}\left[1-\frac{\sqrt{2}\,b^{1/4}\,(c+d\,x)}{a^{1/4}}\right]}{2\,\sqrt{2}\,a^{3/4}\,b^{3/4}\,d^3}+$$
$$\frac{\left(\sqrt{a}+\sqrt{b}\,c^2\right)\text{ArcTan}\left[1+\frac{\sqrt{2}\,b^{1/4}\,(c+d\,x)}{a^{1/4}}\right]}{2\,\sqrt{2}\,a^{3/4}\,b^{3/4}\,d^3}+$$
$$\frac{\left(\sqrt{a}-\sqrt{b}\,c^2\right)\text{Log}\left[\sqrt{a}-\sqrt{2}\,a^{1/4}\,b^{1/4}\,(c+d\,x)+\sqrt{b}\,(c+d\,x)^2\right]}{4\,\sqrt{2}\,a^{3/4}\,b^{3/4}\,d^3}-$$
$$\frac{\left(\sqrt{a}-\sqrt{b}\,c^2\right)\text{Log}\left[\sqrt{a}+\sqrt{2}\,a^{1/4}\,b^{1/4}\,(c+d\,x)+\sqrt{b}\,(c+d\,x)^2\right]}{4\,\sqrt{2}\,a^{3/4}\,b^{3/4}\,d^3}$$

Result (type 7, 106 leaves):

$$\frac{1}{4\,b\,d}\text{RootSum}\left[a+b\,c^4+4\,b\,c^3\,d\,\#1+6\,b\,c^2\,d^2\,\#1^2+4\,b\,c\,d^3\,\#1^3+b\,d^4\,\#1^4\,\&,\right.$$
$$\left.\frac{\text{Log}[x-\#1]\,\#1^2}{c^3+3\,c^2\,d\,\#1+3\,c\,d^2\,\#1^2+d^3\,\#1^3}\,\&\right]$$

Problem 112: Result is not expressed in closed-form.

$$\int\frac{x}{a+b\,(c+d\,x)^4}\,dx$$

Optimal (type 3, 261 leaves, 14 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{b}\,(c+d\,x)^2}{\sqrt{a}}\right]}{2\,\sqrt{a}\,\sqrt{b}\,d^2}+\frac{c\,\text{ArcTan}\left[1-\frac{\sqrt{2}\,b^{1/4}\,(c+d\,x)}{a^{1/4}}\right]}{2\,\sqrt{2}\,a^{3/4}\,b^{1/4}\,d^2}-$$
$$\frac{c\,\text{ArcTan}\left[1+\frac{\sqrt{2}\,b^{1/4}\,(c+d\,x)}{a^{1/4}}\right]}{2\,\sqrt{2}\,a^{3/4}\,b^{1/4}\,d^2}+\frac{c\,\text{Log}\left[\sqrt{a}-\sqrt{2}\,a^{1/4}\,b^{1/4}\,(c+d\,x)+\sqrt{b}\,(c+d\,x)^2\right]}{4\,\sqrt{2}\,a^{3/4}\,b^{1/4}\,d^2}-$$
$$\frac{c\,\text{Log}\left[\sqrt{a}+\sqrt{2}\,a^{1/4}\,b^{1/4}\,(c+d\,x)+\sqrt{b}\,(c+d\,x)^2\right]}{4\,\sqrt{2}\,a^{3/4}\,b^{1/4}\,d^2}$$

Result (type 7, 104 leaves):

$$\frac{1}{4\,b\,d}\text{RootSum}\left[a+b\,c^4+4\,b\,c^3\,d\,\#1+6\,b\,c^2\,d^2\,\#1^2+4\,b\,c\,d^3\,\#1^3+b\,d^4\,\#1^4\,\&,\right.$$
$$\left.\frac{\text{Log}[x-\#1]\,\#1}{c^3+3\,c^2\,d\,\#1+3\,c\,d^2\,\#1^2+d^3\,\#1^3}\,\&\right]$$

Problem 114: Result is not expressed in closed-form.

$$\int\frac{1}{x\,\left(a+b\,(c+d\,x)^4\right)}\,dx$$

Optimal (type 3, 393 leaves, 18 steps):

$$-\frac{\sqrt{b}\,c^2\,\text{ArcTan}\left[\frac{\sqrt{b}\,(c+d\,x)^2}{\sqrt{a}}\right]}{2\sqrt{a}\,\left(a+b\,c^4\right)}+\frac{b^{1/4}\,c\,\left(\sqrt{a}+\sqrt{b}\,c^2\right)\,\text{ArcTan}\left[1-\frac{\sqrt{2}\,b^{1/4}\,(c+d\,x)}{a^{1/4}}\right]}{2\sqrt{2}\,a^{3/4}\,\left(a+b\,c^4\right)}-$$
$$\frac{b^{1/4}\,c\,\left(\sqrt{a}+\sqrt{b}\,c^2\right)\,\text{ArcTan}\left[1+\frac{\sqrt{2}\,b^{1/4}\,(c+d\,x)}{a^{1/4}}\right]}{2\sqrt{2}\,a^{3/4}\,\left(a+b\,c^4\right)}+\frac{\text{Log}[x]}{a+b\,c^4}-$$
$$\left(b^{1/4}\,c\,\left(\sqrt{a}-\sqrt{b}\,c^2\right)\,\text{Log}\left[\sqrt{a}-\sqrt{2}\,a^{1/4}\,b^{1/4}\,(c+d\,x)+\sqrt{b}\,(c+d\,x)^2\right]\right)\Big/\left(4\sqrt{2}\,a^{3/4}\,\left(a+b\,c^4\right)\right)+$$
$$\left(b^{1/4}\,c\,\left(\sqrt{a}-\sqrt{b}\,c^2\right)\,\text{Log}\left[\sqrt{a}+\sqrt{2}\,a^{1/4}\,b^{1/4}\,(c+d\,x)+\sqrt{b}\,(c+d\,x)^2\right]\right)\Big/\left(4\sqrt{2}\,a^{3/4}\,\left(a+b\,c^4\right)\right)-$$
$$\frac{\text{Log}\left[a+b\,(c+d\,x)^4\right]}{4\,\left(a+b\,c^4\right)}$$

Result (type 7, 163 leaves):

$$-\frac{1}{4\,\left(a+b\,c^4\right)}\Big(-4\,\text{Log}[x]+\text{RootSum}\Big[a+b\,c^4+4\,b\,c^3\,d\,\#1+6\,b\,c^2\,d^2\,\#1^2+4\,b\,c\,d^3\,\#1^3+b\,d^4\,\#1^4\,\&,$$
$$\left(4\,c^3\,\text{Log}[x-\#1]+6\,c^2\,d\,\text{Log}[x-\#1]\,\#1+4\,c\,d^2\,\text{Log}[x-\#1]\,\#1^2+d^3\,\text{Log}[x-\#1]\,\#1^3\right)\Big/$$
$$\left(c^3+3\,c^2\,d\,\#1+3\,c\,d^2\,\#1^2+d^3\,\#1^3\right)\,\&\Big]\Big)$$

Problem 115: Result is not expressed in closed-form.

$$\int\frac{1}{x^2\,\left(a+b\,(c+d\,x)^4\right)}\,dx$$

Optimal (type 3, 496 leaves, 18 steps):

$$-\frac{1}{\left(a+b\,c^4\right)\,x}-\frac{\sqrt{b}\,c\,\left(a-b\,c^4\right)\,d\,\text{ArcTan}\left[\frac{\sqrt{b}\,(c+d\,x)^2}{\sqrt{a}}\right]}{\sqrt{a}\,\left(a+b\,c^4\right)^2}+$$
$$\frac{b^{1/4}\,\left(\sqrt{a}\,\left(a-3\,b\,c^4\right)+\sqrt{b}\,c^2\,\left(3\,a-b\,c^4\right)\right)\,d\,\text{ArcTan}\left[1-\frac{\sqrt{2}\,b^{1/4}\,(c+d\,x)}{a^{1/4}}\right]}{2\sqrt{2}\,a^{3/4}\,\left(a+b\,c^4\right)^2}-$$
$$\frac{b^{1/4}\,\left(\sqrt{a}\,\left(a-3\,b\,c^4\right)+\sqrt{b}\,c^2\,\left(3\,a-b\,c^4\right)\right)\,d\,\text{ArcTan}\left[1+\frac{\sqrt{2}\,b^{1/4}\,(c+d\,x)}{a^{1/4}}\right]}{2\sqrt{2}\,a^{3/4}\,\left(a+b\,c^4\right)^2}-\frac{4\,b\,c^3\,d\,\text{Log}[x]}{\left(a+b\,c^4\right)^2}-$$
$$\left(b^{1/4}\,\left(\sqrt{a}\,\left(a-3\,b\,c^4\right)-\sqrt{b}\,c^2\,\left(3\,a-b\,c^4\right)\right)\,d\,\text{Log}\left[\sqrt{a}-\sqrt{2}\,a^{1/4}\,b^{1/4}\,(c+d\,x)+\sqrt{b}\,(c+d\,x)^2\right]\right)\Big/$$
$$\left(4\sqrt{2}\,a^{3/4}\,\left(a+b\,c^4\right)^2\right)+$$
$$\left(b^{1/4}\,\left(\sqrt{a}\,\left(a-3\,b\,c^4\right)-\sqrt{b}\,c^2\,\left(3\,a-b\,c^4\right)\right)\,d\,\text{Log}\left[\sqrt{a}+\sqrt{2}\,a^{1/4}\,b^{1/4}\,(c+d\,x)+\sqrt{b}\,(c+d\,x)^2\right]\right)\Big/$$
$$\left(4\sqrt{2}\,a^{3/4}\,\left(a+b\,c^4\right)^2\right)+\frac{b\,c^3\,d\,\text{Log}\left[a+b\,(c+d\,x)^4\right]}{\left(a+b\,c^4\right)^2}$$

Result (type 7, 238 leaves):

$$\frac{1}{4\left(a+b c^4\right)^2 x}\Big(-4\left(a+b c^4+4 b c^3 d x \operatorname{Log}[x]\right)+ d x \operatorname{RootSum}\Big[a+b c^4+4 b c^3 d \#1+6 b c^2 d^2 \#1^2+4 b c d^3 \#1^3+b d^4 \#1^4 \&, \left(-6 a c^2 \operatorname{Log}[x-\#1]+10 b c^6 \operatorname{Log}[x-\#1]-4 a c d \operatorname{Log}[x-\#1] \#1+ 20 b c^5 d \operatorname{Log}[x-\#1] \#1-a d^2 \operatorname{Log}[x-\#1] \#1^2+15 b c^4 d^2 \operatorname{Log}[x-\#1] \#1^2+ 4 b c^3 d^3 \operatorname{Log}[x-\#1] \#1^3\right) \Big/ \left(c^3+3 c^2 d \#1+3 c d^2 \#1^2+d^3 \#1^3\right) \&\Big]\Big)$$

Problem 120: Result is not expressed in closed-form.

$$\int \frac{1}{a+8 x-8 x^2+4 x^3-x^4} \, dx$$

Optimal (type 3, 89 leaves, 4 steps):

$$-\frac{\operatorname{ArcTan}\left[\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right]}{2 \sqrt{4+a} \sqrt{1-\sqrt{4+a}}}+\frac{\operatorname{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right]}{2 \sqrt{4+a} \sqrt{1+\sqrt{4+a}}}$$

Result (type 7, 57 leaves):

$$-\frac{1}{4} \operatorname{RootSum}\left[a+8 \#1-8 \#1^2+4 \#1^3-\#1^4 \&, \frac{\operatorname{Log}[x-\#1]}{-2+4 \#1-3 \#1^2+\#1^3} \&\right]$$

Problem 121: Result is not expressed in closed-form.

$$\int \frac{1}{\left(a+8 x-8 x^2+4 x^3-x^4\right)^2} \, dx$$

Optimal (type 3, 169 leaves, 5 steps):

$$\frac{\left(5+a+(-1+x)^2\right)(-1+x)}{4\left(12+7 a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)}-\frac{\left(10+3 a+\sqrt{4+a}\right) \operatorname{ArcTan}\left[\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right]}{8(3+a)(4+a)^{3/2} \sqrt{1-\sqrt{4+a}}}+\frac{\left(10+3 a-\sqrt{4+a}\right) \operatorname{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right]}{8(3+a)(4+a)^{3/2} \sqrt{1+\sqrt{4+a}}}$$

Result (type 7, 150 leaves):

$$\frac{(-1+x)\left(6+a-2 x+x^2\right)}{4(3+a)(4+a)\left(a-x\left(-8+8 x-4 x^2+x^3\right)\right)}-\frac{1}{16\left(12+7 a+a^2\right)} \operatorname{RootSum}\Big[a+8 \#1-8 \#1^2+4 \#1^3-\#1^4 \&, \frac{12 \operatorname{Log}[x-\#1]+3 a \operatorname{Log}[x-\#1]-2 \operatorname{Log}[x-\#1] \#1+\operatorname{Log}[x-\#1] \#1^2}{-2+4 \#1-3 \#1^2+\#1^3} \&\Big]$$

Problem 122: Result is not expressed in closed-form.

$$\int \frac{1}{\left(a+8x-8x^2+4x^3-x^4\right)^3}\,dx$$

Optimal (type 3, 252 leaves, 6 steps):

$$\frac{\left(5+a+(-1+x)^2\right)(-1+x)}{8\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^2}+\frac{\left((6+a)(25+7a)+6(7+2a)(-1+x)^2\right)(-1+x)}{32(3+a)^2(4+a)^2\left(3+a-2(-1+x)^2-(-1+x)^4\right)}-\frac{3\left(80+7a^2+14\sqrt{4+a}+a\left(47+4\sqrt{4+a}\right)\right)\text{ArcTan}\left[\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right]}{64(3+a)^2(4+a)^{5/2}\sqrt{1-\sqrt{4+a}}}-\frac{3\left(14+4a-\frac{80+47a+7a^2}{\sqrt{4+a}}\right)\text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right]}{64(3+a)^2(4+a)^2\sqrt{1+\sqrt{4+a}}}$$

Result (type 7, 254 leaves):

$$\frac{1}{128}\left(\frac{16(-1+x)\left(6+a-2x+x^2\right)}{(3+a)(4+a)\left(a-x\left(-8+8x-4x^2+x^3\right)\right)^2}+\frac{4(-1+x)\left(7a^2+6\left(32-14x+7x^2\right)+a\left(79-24x+12x^2\right)\right)}{(3+a)^2(4+a)^2\left(a-x\left(-8+8x-4x^2+x^3\right)\right)}-\frac{1}{\left(12+7a+a^2\right)^2}3\,\text{RootSum}\left[a+8\,\#1-8\,\#1^2+4\,\#1^3-\#1^4\,\&,\left(108\,\text{Log}[x-\#1]+55\,a\,\text{Log}[x-\#1]+7a^2\,\text{Log}[x-\#1]-28\,\text{Log}[x-\#1]\,\#1-8\,a\,\text{Log}[x-\#1]\,\#1+14\,\text{Log}[x-\#1]\,\#1^2+4\,a\,\text{Log}[x-\#1]\,\#1^2\right)\Big/\left(-2+4\,\#1-3\,\#1^2+\#1^3\right)\,\&\right]\right)$$

Problem 127: Result is not expressed in closed-form.

$$\int \frac{x}{a+8x-8x^2+4x^3-x^4}\,dx$$

Optimal (type 3, 116 leaves, 8 steps):

$$-\frac{\text{ArcTan}\left[\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right]}{2\sqrt{4+a}\sqrt{1-\sqrt{4+a}}}+\frac{\text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right]}{2\sqrt{4+a}\sqrt{1+\sqrt{4+a}}}+\frac{\text{ArcTanh}\left[\frac{1+(-1+x)^2}{\sqrt{4+a}}\right]}{2\sqrt{4+a}}$$

Result (type 7, 59 leaves):

$$-\frac{1}{4}\,\text{RootSum}\left[a+8\,\#1-8\,\#1^2+4\,\#1^3-\#1^4\,\&,\ \frac{\text{Log}[x-\#1]\,\#1}{-2+4\,\#1-3\,\#1^2+\#1^3}\,\&\right]$$

Problem 128: Result is not expressed in closed-form.

$$\int \frac{x}{\left(a+8\,x-8\,x^2+4\,x^3-x^4\right)^2}\,dx$$

Optimal (type 3, 231 leaves, 10 steps):

$$\frac{1+(-1+x)^2}{4\,(4+a)\left(3+a-2\,(-1+x)^2-(-1+x)^4\right)}+\frac{\left(5+a+(-1+x)^2\right)(-1+x)}{4\,(12+7\,a+a^2)\left(3+a-2\,(-1+x)^2-(-1+x)^4\right)}-\frac{\left(10+3\,a+\sqrt{4+a}\right)\text{ArcTan}\left[\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right]}{8\,(3+a)\,(4+a)^{3/2}\sqrt{1-\sqrt{4+a}}}+\frac{\left(10+3\,a-\sqrt{4+a}\right)\text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right]}{8\,(3+a)\,(4+a)^{3/2}\sqrt{1+\sqrt{4+a}}}+\frac{\text{ArcTanh}\left[\frac{1+(-1+x)^2}{\sqrt{4+a}}\right]}{4\,(4+a)^{3/2}}$$

Result (type 7, 166 leaves):

$$\frac{a+2\,x-a\,x+a\,x^2+x^3}{4\,(3+a)\,(4+a)\left(a-x\left(-8+8\,x-4\,x^2+x^3\right)\right)}-\frac{1}{16\,(12+7\,a+a^2)}\text{RootSum}\Big[a+8\,\#1-8\,\#1^2+4\,\#1^3-\#1^4\,\&,$$
$$\left(6\,\text{Log}[x-\#1]+a\,\text{Log}[x-\#1]+4\,\text{Log}[x-\#1]\,\#1+2\,a\,\text{Log}[x-\#1]\,\#1+\text{Log}[x-\#1]\,\#1^2\right)\Big/$$
$$\left(-2+4\,\#1-3\,\#1^2+\#1^3\right)\,\&\Big]$$

Problem 129: Result is not expressed in closed-form.

$$\int \frac{x}{\left(a+8\,x-8\,x^2+4\,x^3-x^4\right)^3}\,dx$$

Optimal (type 3, 349 leaves, 12 steps):

$$\frac{1+(-1+x)^2}{8\,(4+a)\,\left(3+a-2\,(-1+x)^2-(-1+x)^4\right)^2}+$$
$$\frac{3\,\left(1+(-1+x)^2\right)}{16\,(4+a)^2\,\left(3+a-2\,(-1+x)^2-(-1+x)^4\right)}+\frac{\left(5+a+(-1+x)^2\right)\,(-1+x)}{8\,\left(12+7\,a+a^2\right)\,\left(3+a-2\,(-1+x)^2-(-1+x)^4\right)^2}+$$
$$\frac{\left((6+a)\,(25+7\,a)+6\,(7+2\,a)\,(-1+x)^2\right)\,(-1+x)}{32\,(3+a)^2\,(4+a)^2\,\left(3+a-2\,(-1+x)^2-(-1+x)^4\right)}-$$
$$\frac{3\,\left(80+7\,a^2+14\,\sqrt{4+a}+a\,\left(47+4\,\sqrt{4+a}\right)\right)\,\text{ArcTan}\left[\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right]}{64\,(3+a)^2\,(4+a)^{5/2}\,\sqrt{1-\sqrt{4+a}}}-$$
$$\frac{3\,\left(14+4\,a-\frac{80+47\,a+7\,a^2}{\sqrt{4+a}}\right)\,\text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right]}{64\,(3+a)^2\,(4+a)^2\,\sqrt{1+\sqrt{4+a}}}+\frac{3\,\text{ArcTanh}\left[\frac{1+(-1+x)^2}{\sqrt{4+a}}\right]}{16\,(4+a)^{5/2}}$$

Result (type 7, 284 leaves):

$$\frac{1}{128}\left(\frac{16\,\left(a+2\,x-a\,x+a\,x^2+x^3\right)}{(3+a)\,(4+a)\,\left(a-x\,\left(-8+8\,x-4\,x^2+x^3\right)\right)^2}+\right.$$
$$\left(4\,\left(a^2\,\left(5-5\,x+6\,x^2\right)+6\,\left(-14+28\,x-12\,x^2+7\,x^3\right)+a\,\left(-7+31\,x+12\,x^3\right)\right)\right)\Big/$$
$$\left((3+a)^2\,(4+a)^2\,\left(a-x\,\left(-8+8\,x-4\,x^2+x^3\right)\right)\right)-$$
$$\frac{1}{\left(12+7\,a+a^2\right)^2}3\,\text{RootSum}\Big[a+8\,\#1-8\,\#1^2+4\,\#1^3-\#1^4\,\&,$$
$$\left(72\,\text{Log}[x-\#1]+31\,a\,\text{Log}[x-\#1]+3\,a^2\,\text{Log}[x-\#1]+8\,\text{Log}[x-\#1]\,\#1+16\,a\,\text{Log}[x-\#1]\,\#1+\right.$$
$$\left.\left.4\,a^2\,\text{Log}[x-\#1]\,\#1+14\,\text{Log}[x-\#1]\,\#1^2+4\,a\,\text{Log}[x-\#1]\,\#1^2\right)\Big/\left(-2+4\,\#1-3\,\#1^2+\#1^3\right)\,\&\Big]\right)$$

Problem 134: Result is not expressed in closed-form.

$$\int\frac{x^2}{a+8\,x-8\,x^2+4\,x^3-x^4}\,dx$$

Optimal (type 3, 99 leaves, 9 steps):

$$-\frac{\text{ArcTan}\left[\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right]}{2\,\sqrt{1-\sqrt{4+a}}}-\frac{\text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right]}{2\,\sqrt{1+\sqrt{4+a}}}+\frac{\text{ArcTanh}\left[\frac{1+(-1+x)^2}{\sqrt{4+a}}\right]}{\sqrt{4+a}}$$

Result (type 7, 61 leaves):

$$-\frac{1}{4}\,\text{RootSum}\left[a+8\,\#1-8\,\#1^2+4\,\#1^3-\#1^4\,\&,\,\frac{\text{Log}[x-\#1]\,\#1^2}{-2+4\,\#1-3\,\#1^2+\#1^3}\,\&\right]$$

Problem 135: Result is not expressed in closed-form.

$$\int \frac{x^2}{\left(a+8x-8x^2+4x^3-x^4\right)^2}\,dx$$

Optimal (type 3, 225 leaves, 11 steps):

$$\frac{1+(-1+x)^2}{2\,(4+a)\left(3+a-2\,(-1+x)^2-(-1+x)^4\right)}+\frac{(4+a)\left(2+(-1+x)^2\right)(-1+x)}{4\left(12+7a+a^2\right)\left(3+a-2\,(-1+x)^2-(-1+x)^4\right)}-\frac{\left(4+a+\sqrt{4+a}\right)\text{ArcTan}\left[\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right]}{8\,(3+a)\,(4+a)\sqrt{1-\sqrt{4+a}}}-\frac{\left(4+a-\sqrt{4+a}\right)\text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right]}{8\,(3+a)\,(4+a)\sqrt{1+\sqrt{4+a}}}+\frac{\text{ArcTanh}\left[\frac{1+(-1+x)^2}{\sqrt{4+a}}\right]}{2\,(4+a)^{3/2}}$$

Result (type 7, 182 leaves):

$$\frac{2x\left(4-3x+2x^2\right)+a\left(1+x-x^2+x^3\right)}{4\,(3+a)\,(4+a)\left(a-x\left(-8+8x-4x^2+x^3\right)\right)}-\frac{1}{16\left(12+7a+a^2\right)}\text{RootSum}\Big[a+8\,\#1-8\,\#1^2+4\,\#1^3-\#1^4\,\&,$$
$$\left(-a\,\text{Log}[x-\#1]+4\,\text{Log}[x-\#1]\,\#1+2\,a\,\text{Log}[x-\#1]\,\#1+4\,\text{Log}[x-\#1]\,\#1^2+a\,\text{Log}[x-\#1]\,\#1^2\right)\Big/$$
$$\left(-2+4\,\#1-3\,\#1^2+\#1^3\right)\,\&\Big]$$

Problem 136: Result is not expressed in closed-form.

$$\int \frac{x^4}{27a^3+27a^2b\,x^2+27a^2c\,x^3+9ab^2x^4+b^3x^6}\,dx$$

Optimal (type 3, 545 leaves, 14 steps):

$$-\left(\left((-1)^{1/3}\left(2(-1)^{1/3}b+3a^{1/3}c^{2/3}\right)\text{ArcTan}\left[\frac{3(-1)^{1/3}a^{2/3}c^{1/3}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}a^{1/3}c^{2/3}}}\right]\right)\Big/\left(3\sqrt{3}\left(1+(-1)^{1/3}\right)^2a^{5/6}b^2\sqrt{4b-3(-1)^{2/3}a^{1/3}c^{2/3}}\,c^{2/3}\right)\right)-$$

$$\frac{\left(2b-3a^{1/3}c^{2/3}\right)\text{ArcTan}\left[\frac{3a^{2/3}c^{1/3}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3a^{1/3}c^{2/3}}}\right]}{9\sqrt{3}a^{5/6}b^2\sqrt{4b-3a^{1/3}c^{2/3}}\,c^{2/3}}-$$

$$\left((-1)^{2/3}\left(2b+3(-1)^{1/3}a^{1/3}c^{2/3}\right)\text{ArcTan}\left[\frac{3(-1)^{2/3}a^{2/3}c^{1/3}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b+3(-1)^{1/3}a^{1/3}c^{2/3}}}\right]\right)\Big/$$

$$\left(3\sqrt{3}\left(1-(-1)^{1/3}\right)\left(1+(-1)^{1/3}\right)^2a^{5/6}b^2\sqrt{4b+3(-1)^{1/3}a^{1/3}c^{2/3}}\,c^{2/3}\right)-$$

$$\frac{\text{Log}\left[3a+3a^{2/3}c^{1/3}x+bx^2\right]}{18a^{2/3}b^2c^{1/3}}+\frac{\text{Log}\left[3a-3(-1)^{1/3}a^{2/3}c^{1/3}x+bx^2\right]}{6\left(1+(-1)^{1/3}\right)^2a^{2/3}b^2c^{1/3}}+$$

$$\frac{(-1)^{1/3}\text{Log}\left[3a+3(-1)^{2/3}a^{2/3}c^{1/3}x+bx^2\right]}{18a^{2/3}b^2c^{1/3}}$$

Result (type 7, 99 leaves) :

$$\frac{1}{3}\text{RootSum}\Big[27a^3+27a^2b\,\sharp1^2+27a^2c\,\sharp1^3+9ab^2\,\sharp1^4+b^3\,\sharp1^6\,\&,$$

$$\frac{\text{Log}[x-\sharp1]\,\sharp1^3}{18a^2b+27a^2c\,\sharp1+12ab^2\,\sharp1^2+2b^3\,\sharp1^4}\,\&\Big]$$

Problem 137: Result is not expressed in closed-form.

$$\int\frac{x^3}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$$

Optimal (type 3, 487 leaves, 14 steps) :

$$-\frac{\text{ArcTan}\left[\frac{3(-1)^{1/3}a^{2/3}c^{1/3}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}a^{1/3}c^{2/3}}}\right]}{3\sqrt{3}\left(1+(-1)^{1/3}\right)^2 a^{7/6}b\sqrt{4b-3(-1)^{2/3}a^{1/3}c^{2/3}}\,c^{1/3}}-$$

$$\frac{\text{ArcTan}\left[\frac{3a^{2/3}c^{1/3}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3a^{1/3}c^{2/3}}}\right]}{9\sqrt{3}\,a^{7/6}b\sqrt{4b-3a^{1/3}c^{2/3}}\,c^{1/3}}+\left((-1)^{1/3}\text{ArcTan}\left[\frac{3(-1)^{2/3}a^{2/3}c^{1/3}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b+3(-1)^{1/3}a^{1/3}c^{2/3}}}\right]\right)\Big/$$

$$\left(3\sqrt{3}\left(1-(-1)^{1/3}\right)\left(1+(-1)^{1/3}\right)^2a^{7/6}b\sqrt{4b+3(-1)^{1/3}a^{1/3}c^{2/3}}\,c^{1/3}\right)+$$

$$\frac{\text{Log}\left[3a+3a^{2/3}c^{1/3}x+bx^2\right]}{54a^{4/3}bc^{2/3}}-\frac{(-1)^{2/3}\text{Log}\left[3a-3(-1)^{1/3}a^{2/3}c^{1/3}x+bx^2\right]}{18\left(1+(-1)^{1/3}\right)^2a^{4/3}bc^{2/3}}+$$

$$\frac{(-1)^{2/3}\text{Log}\left[3a+3(-1)^{2/3}a^{2/3}c^{1/3}x+bx^2\right]}{54a^{4/3}bc^{2/3}}$$

Result (type 7, 99 leaves):

$$\frac{1}{3}\text{RootSum}\left[27a^3+27a^2b\,\#1^2+27a^2c\,\#1^3+9ab^2\,\#1^4+b^3\,\#1^6\,\&,\right.$$
$$\left.\frac{\text{Log}[x-\#1]\,\#1^2}{18a^2b+27a^2c\,\#1+12ab^2\,\#1^2+2b^3\,\#1^4}\,\&\right]$$

Problem 138: Result is not expressed in closed-form.

$$\int\frac{x^2}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$$

Optimal (type 3, 334 leaves, 8 steps):

$$\frac{2(-1)^{2/3}\text{ArcTan}\left[\frac{3(-1)^{1/3}a^{2/3}c^{1/3}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}a^{1/3}c^{2/3}}}\right]}{9\sqrt{3}\left(1+(-1)^{1/3}\right)^2a^{11/6}\sqrt{4b-3(-1)^{2/3}a^{1/3}c^{2/3}}\,c^{2/3}}+\frac{2\,\text{ArcTan}\left[\frac{3a^{2/3}c^{1/3}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3a^{1/3}c^{2/3}}}\right]}{27\sqrt{3}\,a^{11/6}\sqrt{4b-3a^{1/3}c^{2/3}}\,c^{2/3}}+$$

$$\frac{2(-1)^{2/3}\text{ArcTan}\left[\frac{3(-1)^{2/3}a^{2/3}c^{1/3}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b+3(-1)^{1/3}a^{1/3}c^{2/3}}}\right]}{9\sqrt{3}\left(1-(-1)^{1/3}\right)\left(1+(-1)^{1/3}\right)^2a^{11/6}\sqrt{4b+3(-1)^{1/3}a^{1/3}c^{2/3}}\,c^{2/3}}$$

Result (type 7, 97 leaves):

$$\frac{1}{3}\text{RootSum}\left[27a^3+27a^2b\,\#1^2+27a^2c\,\#1^3+9ab^2\,\#1^4+b^3\,\#1^6\,\&,\right.$$
$$\left.\frac{\text{Log}[x-\#1]\,\#1}{18a^2b+27a^2c\,\#1+12ab^2\,\#1^2+2b^3\,\#1^4}\,\&\right]$$

Problem 139: Result is not expressed in closed-form.

$$\int\frac{x}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$$

Optimal (type 3, 469 leaves, 14 steps):

$$-\frac{\text{ArcTan}\left[\frac{3(-1)^{1/3}a^{2/3}c^{1/3}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}a^{1/3}c^{2/3}}}\right]}{9\sqrt{3}\left(1+(-1)^{1/3}\right)^2 a^{13/6}\sqrt{4b-3(-1)^{2/3}a^{1/3}c^{2/3}}\ c^{1/3}}-\frac{\text{ArcTan}\left[\frac{3a^{2/3}c^{1/3}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3a^{1/3}c^{2/3}}}\right]}{27\sqrt{3}\ a^{13/6}\sqrt{4b-3a^{1/3}c^{2/3}}\ c^{1/3}}+$$

$$\frac{(-1)^{1/3}\text{ArcTan}\left[\frac{3(-1)^{2/3}a^{2/3}c^{1/3}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b+3(-1)^{1/3}a^{1/3}c^{2/3}}}\right]}{9\sqrt{3}\left(1-(-1)^{1/3}\right)\left(1+(-1)^{1/3}\right)^2 a^{13/6}\sqrt{4b+3(-1)^{1/3}a^{1/3}c^{2/3}}\ c^{1/3}}-$$

$$\frac{\text{Log}\left[3a+3a^{2/3}c^{1/3}x+bx^2\right]}{162\,a^{7/3}c^{2/3}}+\frac{(-1)^{2/3}\text{Log}\left[3a-3(-1)^{1/3}a^{2/3}c^{1/3}x+bx^2\right]}{54\left(1+(-1)^{1/3}\right)^2 a^{7/3}c^{2/3}}-$$

$$\frac{(-1)^{2/3}\text{Log}\left[3a+3(-1)^{2/3}a^{2/3}c^{1/3}x+bx^2\right]}{162\,a^{7/3}c^{2/3}}$$

Result (type 7, 95 leaves):

$$\frac{1}{3}\text{RootSum}\left[27a^3+27a^2b\,\#1^2+27a^2c\,\#1^3+9ab^2\,\#1^4+b^3\,\#1^6\ \&,\right.$$
$$\left.\frac{\text{Log}[x-\#1]}{18a^2b+27a^2c\,\#1+12ab^2\,\#1^2+2b^3\,\#1^4}\ \&\right]$$

Problem 140: Result is not expressed in closed-form.

$$\int\frac{1}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$$

Optimal (type 3, 522 leaves, 14 steps):

$$-\left(\left((-1)^{1/3}\left(2(-1)^{1/3} b+3 a^{1/3} c^{2/3}\right) \text{ArcTan}\left[\frac{3(-1)^{1/3} a^{2/3} c^{1/3}-2 b x}{\sqrt{3} \sqrt{a} \sqrt{4 b-3(-1)^{2/3} a^{1/3} c^{2/3}}}\right]\right) \Big/ \left(27 \sqrt{3}\left(1+(-1)^{1/3}\right)^2 a^{17/6} \sqrt{4 b-3(-1)^{2/3} a^{1/3} c^{2/3}}\; c^{2/3}\right)\right)-$$

$$\frac{\left(2 b-3 a^{1/3} c^{2/3}\right) \text{ArcTan}\left[\frac{3 a^{2/3} c^{1/3}+2 b x}{\sqrt{3} \sqrt{a} \sqrt{4 b-3 a^{1/3} c^{2/3}}}\right]}{81 \sqrt{3} a^{17/6} \sqrt{4 b-3 a^{1/3} c^{2/3}}\; c^{2/3}}-$$

$$\left(\left(2(-1)^{2/3} b-3 a^{1/3} c^{2/3}\right) \text{ArcTan}\left[\frac{3(-1)^{2/3} a^{2/3} c^{1/3}+2 b x}{\sqrt{3} \sqrt{a} \sqrt{4 b+3(-1)^{1/3} a^{1/3} c^{2/3}}}\right]\right) \Big/ \left(27 \sqrt{3}\left(1-(-1)^{1/3}\right)\left(1+(-1)^{1/3}\right)^2 a^{17/6} \sqrt{4 b+3(-1)^{1/3} a^{1/3} c^{2/3}}\; c^{2/3}\right)+$$

$$\frac{\text{Log}\left[3 a+3 a^{2/3} c^{1/3} x+b x^2\right]}{162 a^{8/3} c^{1/3}}-\frac{\text{Log}\left[3 a-3(-1)^{1/3} a^{2/3} c^{1/3} x+b x^2\right]}{54\left(1+(-1)^{1/3}\right)^2 a^{8/3} c^{1/3}}-$$

$$\frac{(-1)^{1/3} \text{Log}\left[3 a+3(-1)^{2/3} a^{2/3} c^{1/3} x+b x^2\right]}{162 a^{8/3} c^{1/3}}$$

Result (type 7, 99 leaves):

$$\frac{1}{3} \text{RootSum}\Big[27 a^3+27 a^2 b \#1^2+27 a^2 c \#1^3+9 a b^2 \#1^4+b^3 \#1^6 \&, \frac{\text{Log}[x-\#1]}{18 a^2 b \#1+27 a^2 c \#1^2+12 a b^2 \#1^3+2 b^3 \#1^5} \&\Big]$$

Problem 141: Result is not expressed in closed-form.

$$\int \frac{1}{x\left(27 a^3+27 a^2 b x^2+27 a^2 c x^3+9 a b^2 x^4+b^3 x^6\right)} \, dx$$

Optimal (type 3, 563 leaves, 14 steps):

$$\frac{\left(b-(-1)^{2/3} a^{1/3} c^{2/3}\right) \text{ArcTan}\left[\frac{3 (-1)^{1/3} a^{2/3} c^{1/3}-2 b x}{\sqrt{3} \sqrt{a} \sqrt{4 b-3 (-1)^{2/3} a^{1/3} c^{2/3}}}\right]}{9 \sqrt{3} \left(1+(-1)^{1/3}\right)^2 a^{19/6} \sqrt{4 b-3 (-1)^{2/3} a^{1/3} c^{2/3}}\, c^{1/3}}+$$

$$\frac{\left(b-a^{1/3} c^{2/3}\right) \text{ArcTan}\left[\frac{3 a^{2/3} c^{1/3}+2 b x}{\sqrt{3} \sqrt{a} \sqrt{4 b-3 a^{1/3} c^{2/3}}}\right]}{27 \sqrt{3} a^{19/6} \sqrt{4 b-3 a^{1/3} c^{2/3}}\, c^{1/3}}+$$

$$\frac{(-1)^{2/3} \left((-1)^{2/3} b-a^{1/3} c^{2/3}\right) \text{ArcTan}\left[\frac{3 (-1)^{2/3} a^{2/3} c^{1/3}+2 b x}{\sqrt{3} \sqrt{a} \sqrt{4 b+3 (-1)^{1/3} a^{1/3} c^{2/3}}}\right]}{9 \sqrt{3} \left(1-(-1)^{1/3}\right) \left(1+(-1)^{1/3}\right)^2 a^{19/6} \sqrt{4 b+3 (-1)^{1/3} a^{1/3} c^{2/3}}\, c^{1/3}}+$$

$$\frac{\text{Log}[x]}{27 a^3}-\frac{\left(3 a^{1/3}-\frac{b}{c^{2/3}}\right) \text{Log}\left[3 a+3 a^{2/3} c^{1/3} x+b x^2\right]}{486 a^{10/3}}-$$

$$\frac{\left(b+i \sqrt{3} b+6 a^{1/3} c^{2/3}\right) \text{Log}\left[3 a-3 (-1)^{1/3} a^{2/3} c^{1/3} x+b x^2\right]}{972 a^{10/3} c^{2/3}}-$$

$$\frac{\left(3 a^{1/3}-\frac{(-1)^{2/3} b}{c^{2/3}}\right) \text{Log}\left[3 a+3 (-1)^{2/3} a^{2/3} c^{1/3} x+b x^2\right]}{486 a^{10/3}}$$

Result (type 7, 157 leaves):

$$-\frac{1}{81 a^3}\Big(-3 \text{Log}[x]+\text{RootSum}\Big[27 a^3+27 a^2 b \#1^2+27 a^2 c \#1^3+9 a b^2 \#1^4+b^3 \#1^6 \&,$$
$$\left(27 a^2 b \text{Log}[x-\#1]+27 a^2 c \text{Log}[x-\#1] \#1+9 a b^2 \text{Log}[x-\#1] \#1^2+b^3 \text{Log}[x-\#1] \#1^4\right) \Big/$$
$$\left(18 a^2 b+27 a^2 c \#1+12 a b^2 \#1^2+2 b^3 \#1^4\right) \&\Big]\Big)$$

Problem 142: Result is not expressed in closed-form.

$$\int \frac{1}{x^2 \left(27 a^3+27 a^2 b x^2+27 a^2 c x^3+9 a b^2 x^4+b^3 x^6\right)} \, dx$$

Optimal (type 3, 645 leaves, 14 steps):

$$-\frac{1}{27 a^3 x}+$$

$$\left(\left(2 (-1)^{2/3} b^2+12 (-1)^{1/3} a^{1/3} b c^{2/3}+9 a^{2/3} c^{4/3}\right) \text{ArcTan}\left[\frac{3 (-1)^{1/3} a^{2/3} c^{1/3}-2 b x}{\sqrt{3} \sqrt{a} \sqrt{4 b-3 (-1)^{2/3} a^{1/3} c^{2/3}}}\right]\right)\Big/$$

$$\left(81 \sqrt{3} \left(1+(-1)^{1/3}\right)^2 a^{23/6} \sqrt{4 b-3 (-1)^{2/3} a^{1/3} c^{2/3}}\ c^{2/3}\right)+$$

$$\frac{\left(2 b^2-12 a^{1/3} b c^{2/3}+9 a^{2/3} c^{4/3}\right) \text{ArcTan}\left[\frac{3 a^{2/3} c^{1/3}+2 b x}{\sqrt{3} \sqrt{a} \sqrt{4 b-3 a^{1/3} c^{2/3}}}\right]}{243 \sqrt{3} a^{23/6} \sqrt{4 b-3 a^{1/3} c^{2/3}}\ c^{2/3}}+$$

$$\left((-1)^{2/3} \left(2 b^2+12 (-1)^{1/3} a^{1/3} b c^{2/3}+9 (-1)^{2/3} a^{2/3} c^{4/3}\right)\right.$$

$$\left.\text{ArcTan}\left[\frac{3 (-1)^{2/3} a^{2/3} c^{1/3}+2 b x}{\sqrt{3} \sqrt{a} \sqrt{4 b+3 (-1)^{1/3} a^{1/3} c^{2/3}}}\right]\right)\Big/$$

$$\left(81 \sqrt{3} \left(1-(-1)^{1/3}\right) \left(1+(-1)^{1/3}\right)^2 a^{23/6} \sqrt{4 b+3 (-1)^{1/3} a^{1/3} c^{2/3}}\ c^{2/3}\right)-$$

$$\frac{\left(2 b-3 a^{1/3} c^{2/3}\right) \text{Log}\left[3 a+3 a^{2/3} c^{1/3} x+b x^2\right]}{486 a^{11/3} c^{1/3}}+$$

$$\frac{\left(2 b-3 (-1)^{2/3} a^{1/3} c^{2/3}\right) \text{Log}\left[3 a-3 (-1)^{1/3} a^{2/3} c^{1/3} x+b x^2\right]}{162 \left(1+(-1)^{1/3}\right)^2 a^{11/3} c^{1/3}}+$$

$$\frac{(-1)^{1/3} \left(2 b+3 (-1)^{1/3} a^{1/3} c^{2/3}\right) \text{Log}\left[3 a+3 (-1)^{2/3} a^{2/3} c^{1/3} x+b x^2\right]}{486 a^{11/3} c^{1/3}}$$

Result (type 7, 163 leaves):

$$-\frac{1}{81 a^3 x}\Big(3+x\,\text{RootSum}\Big[27 a^3+27 a^2 b \#1^2+27 a^2 c \#1^3+9 a b^2 \#1^4+b^3 \#1^6\ \&,$$
$$\left(27 a^2 b \text{Log}[x-\#1]+27 a^2 c \text{Log}[x-\#1] \#1+9 a b^2 \text{Log}[x-\#1] \#1^2+b^3 \text{Log}[x-\#1] \#1^4\right)\Big/$$
$$\left(18 a^2 b \#1+27 a^2 c \#1^2+12 a b^2 \#1^3+2 b^3 \#1^5\right)\ \&\Big]\Big)$$

Problem 143: Result is not expressed in closed-form.

$$\int \frac{x^5}{216+108 x^2+324 x^3+18 x^4+x^6}\, dx$$

Optimal (type 3, 395 leaves, 14 steps):

$$-\frac{(-2)^{1/3}\left(1+(-2)^{1/3}\,3^{2/3}\right)\text{ArcTan}\left[\frac{3\,(-2)^{2/3}\,3^{1/3}+2\,x}{\sqrt{6\left(4+3\,(-2)^{1/3}\,3^{2/3}\right)}}\right]}{3^{5/6}\sqrt{8+9\,\mathrm{i}\,2^{1/3}\times 3^{1/6}+3\times 2^{1/3}\times 3^{2/3}}}+$$

$$\frac{\left(\frac{3}{2}\right)^{1/6}\left(1-(-3)^{2/3}\,2^{1/3}\right)\text{ArcTan}\left[\frac{2^{1/6}\left(3\,(-3)^{1/3}-2^{1/3}\,x\right)}{\sqrt{3\left(4-3\,(-3)^{2/3}\,2^{1/3}\right)}}\right]}{\left(1+(-1)^{1/3}\right)^2\sqrt{4-3\,(-3)^{2/3}\,2^{1/3}}}-\frac{\left(1-2^{1/3}\times 3^{2/3}\right)\text{ArcTanh}\left[\frac{2^{1/6}\left(3\times 3^{1/3}+2^{1/3}\,x\right)}{\sqrt{3\left(-4+3\times 2^{1/3}\times 3^{2/3}\right)}}\right]}{2^{1/6}\times 3^{5/6}\sqrt{-4+3\times 2^{1/3}\times 3^{2/3}}}+$$

$$\frac{1}{216}\left(36+2^{2/3}\times 3^{1/3}\left(1+\mathrm{i}\sqrt{3}\right)\right)\text{Log}\left[6-3\,(-3)^{1/3}\,2^{2/3}\,x+x^2\right]+$$

$$\frac{1}{108}\left(18-(-2)^{2/3}\,3^{1/3}\right)\text{Log}\left[6+3\,(-2)^{2/3}\,3^{1/3}\,x+x^2\right]+$$

$$\frac{1}{108}\left(18-2^{2/3}\times 3^{1/3}\right)\text{Log}\left[6+3\times 2^{2/3}\times 3^{1/3}\,x+x^2\right]$$

Result (type 7, 61 leaves):

$$\frac{1}{6}\,\text{RootSum}\left[216+108\,\#1^2+324\,\#1^3+18\,\#1^4+\#1^6\,\&,\ \frac{\text{Log}[x-\#1]\,\#1^4}{36+162\,\#1+12\,\#1^2+\#1^4}\,\&\right]$$

Problem 144: Result is not expressed in closed-form.

$$\int\frac{x^4}{216+108\,x^2+324\,x^3+18\,x^4+x^6}\,dx$$

Optimal (type 3, 377 leaves, 14 steps):

$$\frac{(-1)^{2/3}\left(3\,(-3)^{2/3}-2^{2/3}\right)\text{ArcTan}\left[\frac{3\,(-3)^{1/3}\,2^{2/3}-2\,x}{\sqrt{6\left(4-3\,(-3)^{2/3}\,2^{1/3}\right)}}\right]}{9\times 3^{1/6}\left(1+(-1)^{1/3}\right)^2\sqrt{2\left(4-3\,(-3)^{2/3}\,2^{1/3}\right)}}+$$

$$\frac{\left(9-(-2)^{2/3}\,3^{1/3}\right)\text{ArcTan}\left[\frac{3\,(-2)^{2/3}\,3^{1/3}+2\,x}{\sqrt{6\left(4+3\,(-2)^{1/3}\,3^{2/3}\right)}}\right]}{27\sqrt{3\left(8+9\,\mathrm{i}\,2^{1/3}\times 3^{1/6}+3\times 2^{1/3}\times 3^{2/3}\right)}}-\frac{\left(9-2^{2/3}\times 3^{1/3}\right)\text{ArcTanh}\left[\frac{2^{1/6}\left(3\times 3^{1/3}+2^{1/3}\,x\right)}{\sqrt{3\left(-4+3\times 2^{1/3}\times 3^{2/3}\right)}}\right]}{27\sqrt{6\left(-4+3\times 2^{1/3}\times 3^{2/3}\right)}}+$$

$$\frac{\text{Log}\left[6-3\,(-3)^{1/3}\,2^{2/3}\,x+x^2\right]}{6\times 2^{2/3}\times 3^{1/3}\left(1+(-1)^{1/3}\right)^2}+\frac{\left(-\frac{1}{3}\right)^{1/3}\text{Log}\left[6+3\,(-2)^{2/3}\,3^{1/3}\,x+x^2\right]}{18\times 2^{2/3}}-\frac{\text{Log}\left[6+3\times 2^{2/3}\times 3^{1/3}\,x+x^2\right]}{18\times 2^{2/3}\times 3^{1/3}}$$

Result (type 7, 61 leaves):

$$\frac{1}{6}\,\text{RootSum}\left[216+108\,\#1^2+324\,\#1^3+18\,\#1^4+\#1^6\,\&,\ \frac{\text{Log}[x-\#1]\,\#1^3}{36+162\,\#1+12\,\#1^2+\#1^4}\,\&\right]$$

Problem 145: Result is not expressed in closed-form.

$$\int \frac{x^3}{216+108 x^2+324 x^3+18 x^4+x^6}\, dx$$

Optimal (type 3, 361 leaves, 14 steps):

$$-\frac{\text{ArcTan}\left[\frac{3 (-3)^{1/3} 2^{2/3}-2 x}{\sqrt{6 \left(4-3 (-3)^{2/3} 2^{1/3}\right)}}\right]}{6\times 2^{1/6}\times 3^{5/6} \left(1+(-1)^{1/3}\right)^2 \sqrt{4-3 (-3)^{2/3} 2^{1/3}}}+\frac{(-1)^{1/3} \text{ArcTan}\left[\frac{3 (-2)^{2/3} 3^{1/3}+2 x}{\sqrt{6 \left(4+3 (-2)^{1/3} 3^{2/3}\right)}}\right]}{9\times 2^{2/3}\times 3^{5/6} \sqrt{8+9 i\, 2^{1/3}\times 3^{1/6}+3\times 2^{1/3}\times 3^{2/3}}}+$$

$$\frac{\text{ArcTanh}\left[\frac{2^{1/6} \left(3\times 3^{1/3}+2^{1/3} x\right)}{\sqrt{3 \left(-4+3\times 2^{1/3}\times 3^{2/3}\right)}}\right]}{18\times 2^{1/6}\times 3^{5/6} \sqrt{-4+3\times 2^{1/3}\times 3^{2/3}}}-\frac{(-1)^{2/3} \text{Log}\left[6-3 (-3)^{1/3} 2^{2/3} x+x^2\right]}{36\times 2^{1/3}\times 3^{2/3} \left(1+(-1)^{1/3}\right)^2}+$$

$$\frac{(-1)^{2/3} \text{Log}\left[6+3 (-2)^{2/3} 3^{1/3} x+x^2\right]}{108\times 2^{1/3}\times 3^{2/3}}+\frac{\text{Log}\left[6+3\times 2^{2/3}\times 3^{1/3} x+x^2\right]}{108\times 2^{1/3}\times 3^{2/3}}$$

Result (type 7, 61 leaves):

$$\frac{1}{6} \text{RootSum}\left[216+108 \#1^2+324 \#1^3+18 \#1^4+\#1^6\&,\ \frac{\text{Log}[x-\#1] \#1^2}{36+162 \#1+12 \#1^2+\#1^4}\&\right]$$

Problem 146: Result is not expressed in closed-form.

$$\int \frac{x^2}{216+108 x^2+324 x^3+18 x^4+x^6}\, dx$$

Optimal (type 3, 248 leaves, 8 steps):

$$\frac{(-1)^{2/3} \text{ArcTan}\left[\frac{3 (-3)^{1/3} 2^{2/3}-2 x}{\sqrt{6 \left(4-3 (-3)^{2/3} 2^{1/3}\right)}}\right]}{27\times 2^{5/6}\times 3^{1/6} \left(1+(-1)^{1/3}\right)^2 \sqrt{4-3 (-3)^{2/3} 2^{1/3}}}+$$

$$\frac{(-1)^{2/3} \text{ArcTan}\left[\frac{3 (-2)^{2/3} 3^{1/3}+2 x}{\sqrt{6 \left(4+3 (-2)^{1/3} 3^{2/3}\right)}}\right]}{81\times 2^{1/3}\times 3^{1/6} \sqrt{8+9 i\, 2^{1/3}\times 3^{1/6}+3\times 2^{1/3}\times 3^{2/3}}}-\frac{\text{ArcTanh}\left[\frac{2^{1/6} \left(3\times 3^{1/3}+2^{1/3} x\right)}{\sqrt{3 \left(-4+3\times 2^{1/3}\times 3^{2/3}\right)}}\right]}{81\times 2^{5/6}\times 3^{1/6} \sqrt{-4+3\times 2^{1/3}\times 3^{2/3}}}$$

Result (type 7, 59 leaves):

$$\frac{1}{6} \text{RootSum}\left[216+108 \#1^2+324 \#1^3+18 \#1^4+\#1^6\&,\ \frac{\text{Log}[x-\#1] \#1}{36+162 \#1+12 \#1^2+\#1^4}\&\right]$$

Problem 147: Result is not expressed in closed-form.

$$\int \frac{x}{216+108 x^2+324 x^3+18 x^4+x^6}\, dx$$

Optimal (type 3, 361 leaves, 14 steps):

$$-\frac{\text{ArcTan}\left[\frac{3\,(-3)^{1/3}\,2^{2/3}-2\,x}{\sqrt{6\,\left(4-3\,(-3)^{2/3}\,2^{1/3}\right)}}\right]}{36\times 2^{1/6}\times 3^{5/6}\,\left(1+(-1)^{1/3}\right)^2\,\sqrt{4-3\,(-3)^{2/3}\,2^{1/3}}}+\frac{(-1)^{1/3}\,\text{ArcTan}\left[\frac{3\,(-2)^{2/3}\,3^{1/3}+2\,x}{\sqrt{6\,\left(4+3\,(-2)^{1/3}\,3^{2/3}\right)}}\right]}{54\times 2^{2/3}\times 3^{5/6}\,\sqrt{8+9\,i\,2^{1/3}\times 3^{1/6}+3\times 2^{1/3}\times 3^{2/3}}}+\frac{\text{ArcTanh}\left[\frac{2^{1/6}\,\left(3\times 3^{1/3}+2^{1/3}\,x\right)}{\sqrt{3\,\left(-4+3\times 2^{1/3}\times 3^{2/3}\right)}}\right]}{108\times 2^{1/6}\times 3^{5/6}\,\sqrt{-4+3\times 2^{1/3}\times 3^{2/3}}}+\frac{(-1)^{2/3}\,\text{Log}\left[6-3\,(-3)^{1/3}\,2^{2/3}\,x+x^2\right]}{216\times 2^{1/3}\times 3^{2/3}\,\left(1+(-1)^{1/3}\right)^2}-\frac{(-1)^{2/3}\,\text{Log}\left[6+3\,(-2)^{2/3}\,3^{1/3}\,x+x^2\right]}{648\times 2^{1/3}\times 3^{2/3}}-\frac{\text{Log}\left[6+3\times 2^{2/3}\times 3^{1/3}\,x+x^2\right]}{648\times 2^{1/3}\times 3^{2/3}}$$

Result (type 7, 57 leaves):

$$\frac{1}{6}\,\text{RootSum}\left[216+108\,\#1^2+324\,\#1^3+18\,\#1^4+\#1^6\,\&,\ \frac{\text{Log}[x-\#1]}{36+162\,\#1+12\,\#1^2+\#1^4}\,\&\right]$$

Problem 148: Result is not expressed in closed-form.

$$\int\frac{1}{216+108\,x^2+324\,x^3+18\,x^4+x^6}\,dx$$

Optimal (type 3, 377 leaves, 14 steps):

$$\frac{(-1)^{2/3}\,\left(3\,(-3)^{2/3}-2^{2/3}\right)\,\text{ArcTan}\left[\frac{3\,(-3)^{1/3}\,2^{2/3}-2\,x}{\sqrt{6\,\left(4-3\,(-3)^{2/3}\,2^{1/3}\right)}}\right]}{324\times 3^{1/6}\,\left(1+(-1)^{1/3}\right)^2\,\sqrt{2\,\left(4-3\,(-3)^{2/3}\,2^{1/3}\right)}}+\frac{\left(9-(-2)^{2/3}\,3^{1/3}\right)\,\text{ArcTan}\left[\frac{3\,(-2)^{2/3}\,3^{1/3}+2\,x}{\sqrt{6\,\left(4+3\,(-2)^{1/3}\,3^{2/3}\right)}}\right]}{972\,\sqrt{3\,\left(8+9\,i\,2^{1/3}\times 3^{1/6}+3\times 2^{1/3}\times 3^{2/3}\right)}}-\frac{\left(9-2^{2/3}\times 3^{1/3}\right)\,\text{ArcTanh}\left[\frac{2^{1/6}\,\left(3\times 3^{1/3}+2^{1/3}\,x\right)}{\sqrt{3\,\left(-4+3\times 2^{1/3}\times 3^{2/3}\right)}}\right]}{972\,\sqrt{6\,\left(-4+3\times 2^{1/3}\times 3^{2/3}\right)}}-\frac{\text{Log}\left[6-3\,(-3)^{1/3}\,2^{2/3}\,x+x^2\right]}{216\times 2^{2/3}\times 3^{1/3}\,\left(1+(-1)^{1/3}\right)^2}-\frac{\left(-\frac{1}{3}\right)^{1/3}\,\text{Log}\left[6+3\,(-2)^{2/3}\,3^{1/3}\,x+x^2\right]}{648\times 2^{2/3}}+\frac{\text{Log}\left[6+3\times 2^{2/3}\times 3^{1/3}\,x+x^2\right]}{648\times 2^{2/3}\times 3^{1/3}}$$

Result (type 7, 62 leaves):

$$\frac{1}{6}\,\text{RootSum}\left[216+108\,\#1^2+324\,\#1^3+18\,\#1^4+\#1^6\,\&,\ \frac{\text{Log}[x-\#1]}{36\,\#1+162\,\#1^2+12\,\#1^3+\#1^5}\,\&\right]$$

Problem 149: Result is not expressed in closed-form.

$$\int \frac{1}{x\left(216+108 x^2+324 x^3+18 x^4+x^6\right)}\,dx$$

Optimal (type 3, 415 leaves, 14 steps):

$$\frac{(-1)^{2/3}\left((-2)^{2/3}-2\times 3^{2/3}\right)\text{ArcTan}\left[\frac{3(-2)^{2/3}3^{1/3}+2x}{\sqrt{6\left(4+3(-2)^{1/3}3^{2/3}\right)}}\right]}{216\times 2^{1/3}\times 3^{5/6}\sqrt{8+9\,i\,2^{1/3}\times 3^{1/6}+3\times 2^{1/3}\times 3^{2/3}}}-\frac{(-1)^{2/3}\left((-3)^{1/3}+3\times 2^{1/3}\right)\text{ArcTan}\left[\frac{2^{1/6}\left(3(-3)^{1/3}-2^{1/3}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}2^{1/3}\right)}}\right]}{216\times 6^{1/6}\left(1+(-1)^{1/3}\right)^2\sqrt{4-3(-3)^{2/3}2^{1/3}}}-\frac{\left(1-2^{1/3}\times 3^{2/3}\right)\text{ArcTanh}\left[\frac{2^{1/6}\left(3\times 3^{1/3}+2^{1/3}x\right)}{\sqrt{3\left(-4+3\times 2^{1/3}\times 3^{2/3}\right)}}\right]}{216\times 2^{1/6}\times 3^{5/6}\sqrt{-4+3\times 2^{1/3}\times 3^{2/3}}}+\frac{\text{Log}[x]}{216}-\frac{\left(36+2^{2/3}\times 3^{1/3}\left(1+i\sqrt{3}\right)\right)\text{Log}\left[6-3(-3)^{1/3}2^{2/3}x+x^2\right]}{46656}-\frac{\left(18-(-2)^{2/3}3^{1/3}\right)\text{Log}\left[6+3(-2)^{2/3}3^{1/3}x+x^2\right]}{23328}-\frac{\left(18-2^{2/3}\times 3^{1/3}\right)\text{Log}\left[6+3\times 2^{2/3}\times 3^{1/3}x+x^2\right]}{23328}$$

Result (type 7, 103 leaves):

$$\frac{\text{Log}[x]}{216}-\frac{1}{1296}\text{RootSum}\Big[216+108\,\#1^2+324\,\#1^3+18\,\#1^4+\#1^6\,\&,\ \left(108\,\text{Log}[x-\#1]+324\,\text{Log}[x-\#1]\,\#1+18\,\text{Log}[x-\#1]\,\#1^2+\text{Log}[x-\#1]\,\#1^4\right)\Big/\left(36+162\,\#1+12\,\#1^2+\#1^4\right)\,\&\Big]$$

Problem 150: Result is not expressed in closed-form.

$$\int \frac{1}{x^2\left(216+108 x^2+324 x^3+18 x^4+x^6\right)}\,dx$$

Optimal (type 3, 448 leaves, 14 steps):

$$-\frac{1}{216\,x}-\frac{\left(27\,(-6)^{1/3}-(-2)^{2/3}+12\times 3^{2/3}\right)\,\text{ArcTan}\left[\frac{3\,(-2)^{2/3}\,3^{1/3}+2\,x}{\sqrt{6\,\left(4+3\,(-2)^{1/3}\,3^{2/3}\right)}}\right]}{5832\times 3^{1/6}\,\sqrt{8+9\,\mathrm{i}\,2^{1/3}\times 3^{1/6}+3\times 2^{1/3}\times 3^{2/3}}}-$$

$$\frac{(-1)^{2/3}\,\left(6\,(-6)^{2/3}+27\,(-3)^{1/3}-2^{1/3}\right)\,\text{ArcTan}\left[\frac{2^{1/6}\,\left(3\,(-3)^{1/3}-2^{1/3}\,x\right)}{\sqrt{3\,\left(4-3\,(-3)^{2/3}\,2^{1/3}\right)}}\right]}{1944\times 6^{1/6}\,\left(1+(-1)^{1/3}\right)^2\,\sqrt{4-3\,(-3)^{2/3}\,2^{1/3}}}-$$

$$\frac{\left(2^{1/3}+27\times 3^{1/3}-6\times 6^{2/3}\right)\,\text{ArcTanh}\left[\frac{2^{1/6}\,\left(3\times 3^{1/3}+2^{1/3}\,x\right)}{\sqrt{3\,\left(-4+3\times 2^{1/3}\times 3^{2/3}\right)}}\right]}{5832\times 6^{1/6}\,\sqrt{-4+3\times 2^{1/3}\times 3^{2/3}}}-$$

$$\frac{(-1)^{2/3}\,\left(9+(-3)^{1/3}\,2^{2/3}\right)\,\text{Log}\left[6-3\,(-3)^{1/3}\,2^{2/3}\,x+x^2\right]}{1296\times 2^{1/3}\times 3^{2/3}\,\left(1+(-1)^{1/3}\right)^2}+$$

$$\frac{\left(3\,(-6)^{2/3}+2\,(-2)^{1/3}\right)\,\text{Log}\left[6+3\,(-2)^{2/3}\,3^{1/3}\,x+x^2\right]}{7776\times 3^{1/3}}-\frac{\left(2^{2/3}-3\times 3^{2/3}\right)\,\text{Log}\left[6+3\times 2^{2/3}\times 3^{1/3}\,x+x^2\right]}{3888\times 6^{1/3}}$$

Result (type 7, 109 leaves):

$$-\frac{1}{216\,x}-\frac{1}{1296}\text{RootSum}\Big[216+108\,\#1^2+324\,\#1^3+18\,\#1^4+\#1^6\,\&,$$
$$\left(108\,\text{Log}[x-\#1]+324\,\text{Log}[x-\#1]\,\#1+18\,\text{Log}[x-\#1]\,\#1^2+\text{Log}[x-\#1]\,\#1^4\right)\Big/$$
$$\left(36\,\#1+162\,\#1^2+12\,\#1^3+\#1^5\right)\,\&\Big]$$

Problem 151: Result is not expressed in closed-form.

$$\int\frac{x^8}{\left(216+108\,x^2+324\,x^3+18\,x^4+x^6\right)^2}\,dx$$

Optimal (type 3, 1064 leaves, 23 steps):

$$-\left(\left(\left(-\frac{1}{3}\right)^{1/3}\left(9\left(6+(-3)^{1/3}\,2^{2/3}\right)+\left(2-2^{2/3}\left(6\,(-6)^{2/3}+27\,(-3)^{1/3}\right)\right)x\right)\right)\Big/\left(162\times 2^{2/3}\left(1+(-1)^{1/3}\right)^4\left(4-3\,(-3)^{2/3}\,2^{1/3}\right)\left(6-3\,(-3)^{1/3}\,2^{2/3}\,x+x^2\right)\right)\right)-$$

$$\left(\left(-\frac{1}{3}\right)^{1/3}\left(9\left(6-(-2)^{2/3}\,3^{1/3}\right)+\left(2+27\,(-2)^{2/3}\,3^{1/3}+12\,(-2)^{1/3}\,3^{2/3}\right)x\right)\right)\Big/\left(729\times 2^{2/3}\left(8+9\,i\,2^{1/3}\times 3^{1/6}+3\times 2^{1/3}\times 3^{2/3}\right)\left(6+3\,(-2)^{2/3}\,3^{1/3}\,x+x^2\right)\right)+$$

$$\frac{9\left(6-2^{2/3}\times 3^{1/3}\right)+\left(2+2^{2/3}\left(27\times 3^{1/3}-6\times 6^{2/3}\right)\right)x}{1458\times 2^{2/3}\times 3^{1/3}\left(4-3\times 2^{1/3}\times 3^{2/3}\right)\left(6+3\times 2^{2/3}\times 3^{1/3}\,x+x^2\right)}-$$

$$\frac{i\left((-2)^{2/3}+6\times 3^{2/3}\right)\mathrm{ArcTan}\left[\frac{3\,(-2)^{2/3}\,3^{1/3}+2\,x}{\sqrt{6\left(4+3\,(-2)^{1/3}\,3^{2/3}\right)}}\right]}{162\times 2^{5/6}\times 3^{1/3}\left(1+(-1)^{1/3}\right)^5\sqrt{4+3\,(-2)^{1/3}\,3^{2/3}}}-$$

$$\left((-1)^{1/3}\left(2+27\,(-2)^{2/3}\,3^{1/3}+12\,(-2)^{1/3}\,3^{2/3}\right)\mathrm{ArcTan}\left[\frac{3\,(-2)^{2/3}\,3^{1/3}+2\,x}{\sqrt{6\left(4+3\,(-2)^{1/3}\,3^{2/3}\right)}}\right]\right)\Big/\left(162\times 2^{1/6}\times 3^{5/6}\left(1-(-1)^{1/3}\right)^2\left(1+(-1)^{1/3}\right)^4\left(4+3\,(-2)^{1/3}\,3^{2/3}\right)^{3/2}\right)-$$

$$\frac{(-1)^{1/3}\left(6\,(-6)^{2/3}+27\,(-3)^{1/3}-2^{1/3}\right)\mathrm{ArcTan}\left[\frac{2^{1/6}\left(3\,(-3)^{1/3}-2^{1/3}\,x\right)}{\sqrt{3\left(4-3\,(-3)^{2/3}\,2^{1/3}\right)}}\right]}{81\sqrt{2}\,3^{5/6}\left(1+(-1)^{1/3}\right)^4\left(4-3\,(-3)^{2/3}\,2^{1/3}\right)^{3/2}}+$$

$$\frac{\left(i\,2^{2/3}-9\times 3^{1/6}-3\,i\,3^{2/3}\right)\mathrm{ArcTan}\left[\frac{2^{1/6}\left(3\,(-3)^{1/3}-2^{1/3}\,x\right)}{\sqrt{3\left(4-3\,(-3)^{2/3}\,2^{1/3}\right)}}\right]}{162\times 2^{5/6}\times 3^{1/3}\left(1+(-1)^{1/3}\right)^5\sqrt{4-3\,(-3)^{2/3}\,2^{1/3}}}-$$

$$\frac{\left(1+3\times 2^{1/3}\times 3^{2/3}\right)\mathrm{ArcTanh}\left[\frac{2^{1/6}\left(3\times 3^{1/3}+2^{1/3}\,x\right)}{\sqrt{3\left(-4+3\times 2^{1/3}\times 3^{2/3}\right)}}\right]}{1458\times 2^{1/6}\times 3^{5/6}\sqrt{-4+3\times 2^{1/3}\times 3^{2/3}}}+$$

$$\frac{\left(2^{1/3}+27\times 3^{1/3}-6\times 6^{2/3}\right)\mathrm{ArcTanh}\left[\frac{2^{1/6}\left(3\times 3^{1/3}+2^{1/3}\,x\right)}{\sqrt{3\left(-4+3\times 2^{1/3}\times 3^{2/3}\right)}}\right]}{81\sqrt{2}\,3^{5/6}\left(1-(-1)^{1/3}\right)^2\left(1+(-1)^{1/3}\right)^4\left(-4+3\times 2^{1/3}\times 3^{2/3}\right)^{3/2}}-$$

$$\frac{\mathrm{Log}\left[6-3\,(-3)^{1/3}\,2^{2/3}\,x+x^2\right]}{972\times 2^{1/3}\times 3^{2/3}\left(1+(-1)^{1/3}\right)^4}+\frac{i\,\mathrm{Log}\left[6+3\,(-2)^{2/3}\,3^{1/3}\,x+x^2\right]}{972\times 2^{1/3}\times 3^{1/6}\left(1+(-1)^{1/3}\right)^5}-\frac{\mathrm{Log}\left[6+3\times 2^{2/3}\times 3^{1/3}\,x+x^2\right]}{8748\times 2^{1/3}\times 3^{2/3}}$$

Result (type 7, 167 leaves):

$$\frac{-7884+324\,x-3990\,x^2-11\,610\,x^3-203\,x^4-9\,x^5}{34\,182\,\left(216+108\,x^2+324\,x^3+18\,x^4+x^6\right)}-$$
$$\frac{1}{205\,092}\text{RootSum}\Big[216+108\,\#1^2+324\,\#1^3+18\,\#1^4+\#1^6\,\&,$$
$$\left(324\,\text{Log}[x-\#1]-96\,\text{Log}[x-\#1]\,\#1+324\,\text{Log}[x-\#1]\,\#1^2+406\,\text{Log}[x-\#1]\,\#1^3+\right.$$
$$\left.9\,\text{Log}[x-\#1]\,\#1^4\right)\Big/\left(36\,\#1+162\,\#1^2+12\,\#1^3+\#1^5\right)\,\&\Big]$$

Problem 152: Result is not expressed in closed-form.

$$\int\frac{x^7}{\left(216+108\,x^2+324\,x^3+18\,x^4+x^6\right)^2}\,dx$$

Optimal (type 3, 1005 leaves, 23 steps) :

$$-\left(\left(2\left(2(-1)^{1/3}3^{2/3}+9\times 6^{1/3}\right)-9\left((-2)^{2/3}+2(-1)^{1/3}3^{2/3}\right)x\right)\Big/\right.$$
$$\left.\left(972\times 2^{2/3}\left(1+(-1)^{1/3}\right)^4\left(4-3(-3)^{2/3}2^{1/3}\right)\left(6-3(-3)^{1/3}2^{2/3}x+x^2\right)\right)\right)-$$
$$\frac{(-6)^{1/3}\left(9(-2)^{1/3}+2\times 3^{1/3}\right)-9\left(1+(-2)^{1/3}3^{2/3}\right)x}{4374\left(8+9\,\mathrm{i}\,2^{1/3}\times 3^{1/6}+3\times 2^{1/3}\times 3^{2/3}\right)\left(6+3(-2)^{2/3}3^{1/3}x+x^2\right)}+$$
$$\frac{2\left(2-3\times 2^{1/3}\times 3^{2/3}\right)-3\left(6-2^{2/3}\times 3^{1/3}\right)x}{2916\times 2^{2/3}\times 3^{1/3}\left(4-3\times 2^{1/3}\times 3^{2/3}\right)\left(6+3\times 2^{2/3}\times 3^{1/3}x+x^2\right)}+$$
$$\left(\left(9\,\mathrm{i}+3^{1/3}\left(2\,\mathrm{i}\,2^{2/3}-9\times 3^{1/6}+2\times 2^{2/3}\sqrt{3}\right)\right)\mathrm{ArcTan}\left[\frac{3(-3)^{1/3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}2^{1/3}\right)}}\right]\right)\Big/$$
$$\left(5832\left(1+(-1)^{1/3}\right)^5\sqrt{2\left(4-3(-3)^{2/3}2^{1/3}\right)}\right)+$$
$$\frac{\left(1+(-2)^{1/3}3^{2/3}\right)\mathrm{ArcTan}\left[\frac{3(-2)^{2/3}3^{1/3}+2x}{\sqrt{6\left(4+3(-2)^{1/3}3^{2/3}\right)}}\right]}{54\sqrt{6}\left(1-(-1)^{1/3}\right)^2\left(1+(-1)^{1/3}\right)^4\left(4+3(-2)^{1/3}3^{2/3}\right)^{3/2}}+$$
$$\frac{\left(9\times 3^{1/6}+\mathrm{i}\left(4\times 2^{2/3}-3\times 3^{2/3}\right)\right)\mathrm{ArcTan}\left[\frac{3(-2)^{2/3}3^{1/3}+2x}{\sqrt{6\left(4+3(-2)^{1/3}3^{2/3}\right)}}\right]}{1944\times 3^{2/3}\left(1+(-1)^{1/3}\right)^5\sqrt{2\left(4+3(-2)^{1/3}3^{2/3}\right)}}-$$
$$\frac{(-1)^{1/3}\left((-3)^{1/3}+3\times 2^{1/3}\right)\mathrm{ArcTan}\left[\frac{2^{1/6}\left(3(-3)^{1/3}-2^{1/3}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}2^{1/3}\right)}}\right]}{54\sqrt{2}\,3^{5/6}\left(1+(-1)^{1/3}\right)^4\left(4-3(-3)^{2/3}2^{1/3}\right)^{3/2}}+$$
$$\frac{\left(1-2^{1/3}\times 3^{2/3}\right)\mathrm{ArcTanh}\left[\frac{2^{1/6}\left(3\times 3^{1/3}+2^{1/3}x\right)}{\sqrt{3\left(-4+3\times 2^{1/3}\times 3^{2/3}\right)}}\right]}{54\sqrt{6}\left(1-(-1)^{1/3}\right)^2\left(1+(-1)^{1/3}\right)^4\left(-4+3\times 2^{1/3}\times 3^{2/3}\right)^{3/2}}+$$
$$\frac{\left(2\times 2^{2/3}+3\times 3^{2/3}\right)\mathrm{ArcTanh}\left[\frac{2^{1/6}\left(3\times 3^{1/3}+2^{1/3}x\right)}{\sqrt{3\left(-4+3\times 2^{1/3}\times 3^{2/3}\right)}}\right]}{26\,244\times 3^{1/6}\sqrt{2\left(-4+3\times 2^{1/3}\times 3^{2/3}\right)}}+\frac{\mathrm{i}\,\mathrm{Log}\left[6-3(-3)^{1/3}2^{2/3}x+x^2\right]}{648\times 2^{2/3}\times 3^{5/6}\left(1+(-1)^{1/3}\right)^5}-$$
$$\frac{\left(\mathrm{i}+\sqrt{3}\right)\mathrm{Log}\left[6+3(-2)^{2/3}3^{1/3}x+x^2\right]}{1296\times 2^{2/3}\times 3^{5/6}\left(1+(-1)^{1/3}\right)^5}-\frac{\mathrm{Log}\left[6+3\times 2^{2/3}\times 3^{1/3}x+x^2\right]}{17\,496\times 2^{2/3}\times 3^{1/3}}$$

Result (type 7, 167 leaves):

$$\frac{648-96x+432x^2+908x^3-18x^4+73x^5}{68364\,(216+108x^2+324x^3+18x^4+x^6)}+\frac{1}{410184}\text{RootSum}\Big[216+108\,\#1^2+324\,\#1^3+18\,\#1^4+\#1^6\,\&,\ (96\,\text{Log}[x-\#1]-216\,\text{Log}[x-\#1]\,\#1+96\,\text{Log}[x-\#1]\,\#1^2-36\,\text{Log}[x-\#1]\,\#1^3+73\,\text{Log}[x-\#1]\,\#1^4)\Big/(36\,\#1+162\,\#1^2+12\,\#1^3+\#1^5)\,\&\Big]$$

Problem 153: Result is not expressed in closed-form.

$$\int\frac{x^6}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$$

Optimal (type 3, 677 leaves, 14 steps):

$$\frac{9(-2)^{2/3}+6^{1/3}\left(9+(-3)^{1/3}2^{2/3}\right)x}{2916\times2^{2/3}\left(1+(-1)^{1/3}\right)^4\left(4-3(-3)^{2/3}2^{1/3}\right)\left(6-3(-3)^{1/3}2^{2/3}x+x^2\right)}+\left(9\times2^{2/3}+(-1)^{1/3}3^{2/3}\left(2+3(-2)^{1/3}3^{2/3}\right)x\right)\Big/\left(13122\times2^{2/3}\left(8+9\,i\,2^{1/3}\times3^{1/6}+3\times2^{1/3}\times3^{2/3}\right)\left(6+3(-2)^{2/3}3^{1/3}x+x^2\right)\right)+\frac{3\times2^{2/3}\times3^{1/3}-\left(2-3\times2^{1/3}\times3^{2/3}\right)x}{8748\times2^{2/3}\times3^{1/3}\left(4-3\times2^{1/3}\times3^{2/3}\right)\left(6+3\times2^{2/3}\times3^{1/3}x+x^2\right)}+$$

$$\frac{(-1)^{1/3}\left(3(-3)^{2/3}-2^{2/3}\right)\text{ArcTan}\left[\frac{3(-3)^{1/3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}2^{1/3}\right)}}\right]}{486\times6^{5/6}\left(1+(-1)^{1/3}\right)^4\left(4-3(-3)^{2/3}2^{1/3}\right)^{3/2}}+\frac{\left(3(-3)^{2/3}+(-1)^{1/3}2^{2/3}\right)\text{ArcTan}\left[\frac{3(-2)^{2/3}3^{1/3}+2x}{\sqrt{6\left(4+3(-2)^{1/3}3^{2/3}\right)}}\right]}{486\times6^{5/6}\left(1-(-1)^{1/3}\right)^2\left(1+(-1)^{1/3}\right)^4\left(4+3(-2)^{1/3}3^{2/3}\right)^{3/2}}-$$

$$\frac{\left(2^{2/3}-3\times3^{2/3}\right)\text{ArcTanh}\left[\frac{2^{1/6}\left(3\times3^{1/3}+2^{1/3}x\right)}{\sqrt{3\left(-4+3\times2^{1/3}\times3^{2/3}\right)}}\right]}{486\times6^{5/6}\left(1-(-1)^{1/3}\right)^2\left(1+(-1)^{1/3}\right)^4\left(-4+3\times2^{1/3}\times3^{2/3}\right)^{3/2}}+\frac{\left(-\frac{1}{3}\right)^{1/6}\text{Log}\left[6-3(-3)^{1/3}2^{2/3}x+x^2\right]}{5832\times2^{1/3}\left(1+(-1)^{1/3}\right)^5}-\frac{i\,\text{Log}\left[6+3(-2)^{2/3}3^{1/3}x+x^2\right]}{5832\times2^{1/3}\times3^{1/6}\left(1+(-1)^{1/3}\right)^5}+\frac{\text{Log}\left[6+3\times2^{2/3}\times3^{1/3}x+x^2\right]}{52488\times2^{1/3}\times3^{2/3}}$$

Result (type 7, 167 leaves):

$$\frac{-96+108x-64x^2-72x^3+73x^4-3x^5}{68364\left(216+108x^2+324x^3+18x^4+x^6\right)}-\frac{1}{410184}\text{RootSum}\Big[$$
$$216+108\,\#1^2+324\,\#1^3+18\,\#1^4+\#1^6\,\&,\ \left(108\,\text{Log}[x-\#1]-32\,\text{Log}[x-\#1]\,\#1+108\,\text{Log}[x-\#1]\,\#1^2-\right.$$
$$\left.146\,\text{Log}[x-\#1]\,\#1^3+3\,\text{Log}[x-\#1]\,\#1^4\right)\Big/\left(36\,\#1+162\,\#1^2+12\,\#1^3+\#1^5\right)\,\&\Big]$$

Problem 154: Result is not expressed in closed-form.

$$\int\frac{x^5}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx$$

Optimal (type 3, 682 leaves, 17 steps):

$$\frac{\left(-\frac{1}{3}\right)^{1/3}\left(4-(-3)^{1/3}\,2^{2/3}x\right)}{1944\times 2^{2/3}\left(1+(-1)^{1/3}\right)^4\left(4-3(-3)^{2/3}\,2^{1/3}\right)\left(6-3(-3)^{1/3}\,2^{2/3}x+x^2\right)}+$$
$$\frac{\left(-\frac{1}{3}\right)^{1/3}\left(4+(-2)^{2/3}\,3^{1/3}x\right)}{8748\times 2^{2/3}\left(8+9\,i\,2^{1/3}\times 3^{1/6}+3\times 2^{1/3}\times 3^{2/3}\right)\left(6+3(-2)^{2/3}\,3^{1/3}x+x^2\right)}-$$
$$\frac{4+2^{2/3}\times 3^{1/3}x}{17496\times 2^{2/3}\times 3^{1/3}\left(4-3\times 2^{1/3}\times 3^{2/3}\right)\left(6+3\times 2^{2/3}\times 3^{1/3}x+x^2\right)}-$$
$$\frac{\text{ArcTan}\left[\frac{3(-3)^{1/3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}2^{1/3}\right)}}\right]}{4374\times 2^{5/6}\times 3^{1/6}\left(1+(-1)^{1/3}\right)^4\sqrt{4-3(-3)^{2/3}\,2^{1/3}}}+$$
$$\frac{\text{ArcTan}\left[\frac{3(-3)^{1/3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}2^{1/3}\right)}}\right]}{4374\sqrt{3}\left(8-9\,i\,2^{1/3}\times 3^{1/6}+3\times 2^{1/3}\times 3^{2/3}\right)^{3/2}}-$$
$$\frac{i\,\text{ArcTan}\left[\frac{3(-2)^{2/3}3^{1/3}+2x}{\sqrt{6\left(4+3(-2)^{1/3}3^{2/3}\right)}}\right]}{1458\times 2^{5/6}\times 3^{2/3}\left(1+(-1)^{1/3}\right)^5\sqrt{4+3(-2)^{1/3}\,3^{2/3}}}-$$
$$\frac{\text{ArcTan}\left[\frac{3(-2)^{2/3}3^{1/3}+2x}{\sqrt{6\left(4+3(-2)^{1/3}3^{2/3}\right)}}\right]}{4374\sqrt{3}\left(8+9\,i\,2^{1/3}\times 3^{1/6}+3\times 2^{1/3}\times 3^{2/3}\right)^{3/2}}-$$
$$\frac{\text{ArcTanh}\left[\frac{2^{1/6}\left(3\times 3^{1/3}+2^{1/3}x\right)}{\sqrt{3\left(-4+3\times 2^{1/3}\times 3^{2/3}\right)}}\right]}{8748\sqrt{6}\left(-4+3\times 2^{1/3}\times 3^{2/3}\right)^{3/2}}-\frac{\text{ArcTanh}\left[\frac{2^{1/6}\left(3\times 3^{1/3}+2^{1/3}x\right)}{\sqrt{3\left(-4+3\times 2^{1/3}\times 3^{2/3}\right)}}\right]}{39366\times 2^{5/6}\times 3^{1/6}\sqrt{-4+3\times 2^{1/3}\times 3^{2/3}}}$$

Result (type 7, 167 leaves):

$$\frac{972-144\,x+648\,x^2+729\,x^3-27\,x^4+4\,x^5}{615\,276\,\left(216+108\,x^2+324\,x^3+18\,x^4+x^6\right)}+\frac{1}{3\,691\,656}\text{RootSum}\Big[216+108\,\#1^2+324\,\#1^3+18\,\#1^4+\#1^6\,\&,$$
$$\left(144\,\text{Log}[x-\#1]-324\,\text{Log}[x-\#1]\,\#1+2043\,\text{Log}[x-\#1]\,\#1^2-\right.$$
$$\left.54\,\text{Log}[x-\#1]\,\#1^3+4\,\text{Log}[x-\#1]\,\#1^4\right)\Big/\left(36\,\#1+162\,\#1^2+12\,\#1^3+\#1^5\right)\,\&\Big]$$

Problem 155: Result is not expressed in closed-form.

$$\int\frac{x^4}{\left(216+108\,x^2+324\,x^3+18\,x^4+x^6\right)^2}\,dx$$

Optimal (type 3, 850 leaves, 23 steps):

$$\frac{\left(-\frac{1}{3}\right)^{1/3}\left(3\,(-3)^{1/3}\,2^{2/3}-2\,x\right)}{5832\times 2^{2/3}\left(1+(-1)^{1/3}\right)^4\left(4-3\,(-3)^{2/3}\,2^{1/3}\right)\left(6-3\,(-3)^{1/3}\,2^{2/3}\,x+x^2\right)}-$$

$$\left(\left(-\frac{1}{3}\right)^{1/3}\left(3\,(-2)^{2/3}\,3^{1/3}+2\,x\right)\right)\Big/$$
$$\left(26\,244\times 2^{2/3}\left(8+9\,\mathrm{i}\,2^{1/3}\times 3^{1/6}+3\times 2^{1/3}\times 3^{2/3}\right)\left(6+3\,(-2)^{2/3}\,3^{1/3}\,x+x^2\right)\right)-$$

$$\frac{3\times 3^{1/3}+2^{1/3}\,x}{52\,488\left(9\times 2^{1/3}-4\times 3^{1/3}\right)\left(6+3\times 2^{2/3}\times 3^{1/3}\,x+x^2\right)}+$$

$$\frac{(-1)^{1/3}\,\mathrm{ArcTan}\left[\frac{3\,(-3)^{1/3}\,2^{2/3}-2\,x}{\sqrt{6\left(4-3\,(-3)^{2/3}\,2^{1/3}\right)}}\right]}{729\times 2^{2/3}\times 3^{5/6}\left(1+(-1)^{1/3}\right)^4\left(8-9\,\mathrm{i}\,2^{1/3}\times 3^{1/6}+3\times 2^{1/3}\times 3^{2/3}\right)^{3/2}}-$$

$$\frac{(-1)^{1/3}\,\mathrm{ArcTan}\left[\frac{3\,(-2)^{2/3}\,3^{1/3}+2\,x}{\sqrt{6\left(4+3\,(-2)^{1/3}\,3^{2/3}\right)}}\right]}{2916\times 2^{1/6}\times 3^{5/6}\left(1-(-1)^{1/3}\right)^2\left(1+(-1)^{1/3}\right)^4\left(4+3\,(-2)^{1/3}\,3^{2/3}\right)^{3/2}}-$$

$$\frac{\left(\mathrm{i}+\sqrt{3}\right)\mathrm{ArcTan}\left[\frac{3\,(-2)^{2/3}\,3^{1/3}+2\,x}{\sqrt{6\left(4+3\,(-2)^{1/3}\,3^{2/3}\right)}}\right]}{11\,664\times 2^{1/6}\times 3^{1/3}\left(1+(-1)^{1/3}\right)^5\sqrt{4+3\,(-2)^{1/3}\,3^{2/3}}}-$$

$$\frac{\mathrm{i}\,\mathrm{ArcTan}\left[\frac{2^{1/6}\left(3\,(-3)^{1/3}-2^{1/3}\,x\right)}{\sqrt{3\left(4-3\,(-3)^{2/3}\,2^{1/3}\right)}}\right]}{5832\times 2^{1/6}\times 3^{1/3}\left(1+(-1)^{1/3}\right)^5\sqrt{4-3\,(-3)^{2/3}\,2^{1/3}}}+\frac{\mathrm{ArcTanh}\left[\frac{2^{1/6}\left(3\times 3^{1/3}+2^{1/3}\,x\right)}{\sqrt{3\left(-4+3\times 2^{1/3}\times 3^{2/3}\right)}}\right]}{26\,244\times 2^{1/6}\times 3^{5/6}\left(-4+3\times 2^{1/3}\times 3^{2/3}\right)^{3/2}}+$$

$$\frac{\mathrm{ArcTanh}\left[\frac{2^{1/6}\left(3\times 3^{1/3}+2^{1/3}\,x\right)}{\sqrt{3\left(-4+3\times 2^{1/3}\times 3^{2/3}\right)}}\right]}{52\,488\times 2^{1/6}\times 3^{5/6}\sqrt{-4+3\times 2^{1/3}\times 3^{2/3}}}-\frac{\mathrm{Log}\left[6-3\,(-3)^{1/3}\,2^{2/3}\,x+x^2\right]}{34\,992\times 2^{1/3}\times 3^{2/3}\left(1+(-1)^{1/3}\right)^4}+$$

$$\frac{\mathrm{i}\,\mathrm{Log}\left[6+3\,(-2)^{2/3}\,3^{1/3}\,x+x^2\right]}{34\,992\times 2^{1/3}\times 3^{1/6}\left(1+(-1)^{1/3}\right)^5}-\frac{\mathrm{Log}\left[6+3\times 2^{2/3}\times 3^{1/3}\,x+x^2\right]}{314\,928\times 2^{1/3}\times 3^{2/3}}$$

Result (type 7, 167 leaves):

$$\frac{-288+324\,x-1458\,x^2-216\,x^3+8\,x^4-9\,x^5}{1\,230\,552\left(216+108\,x^2+324\,x^3+18\,x^4+x^6\right)}-\frac{1}{7\,383\,312}\mathrm{RootSum}\left[216+108\,\#1^2+324\,\#1^3+18\,\#1^4+\#1^6\,\&,\right.$$
$$\left(324\,\mathrm{Log}[x-\#1]-2628\,\mathrm{Log}[x-\#1]\,\#1+324\,\mathrm{Log}[x-\#1]\,\#1^2-\right.$$
$$\left.\left.16\,\mathrm{Log}[x-\#1]\,\#1^3+9\,\mathrm{Log}[x-\#1]\,\#1^4\right)\Big/\left(36\,\#1+162\,\#1^2+12\,\#1^3+\#1^5\right)\,\&\right]$$

Problem 156: Result is not expressed in closed-form.

$$\int \frac{x^3}{\left(216 + 108\, x^2 + 324\, x^3 + 18\, x^4 + x^6\right)^2}\, dx$$

Optimal (type 3, 873 leaves, 23 steps) :

$$\frac{(-6)^{1/3}\left(2\,(-3)^{1/3}+9\times 2^{1/3}\right)-3\,x}{157464\,\left(8-9\,i\,2^{1/3}\times 3^{1/6}+3\times 2^{1/3}\times 3^{2/3}\right)\left(6-3\,(-3)^{1/3}\,2^{2/3}\,x+x^2\right)}-$$

$$\frac{(-6)^{1/3}\left(9\,(-2)^{1/3}+2\times 3^{1/3}\right)+3\,x}{157464\,\left(8+9\,i\,2^{1/3}\times 3^{1/6}+3\times 2^{1/3}\times 3^{2/3}\right)\left(6+3\,(-2)^{2/3}\,3^{1/3}\,x+x^2\right)}-$$

$$\frac{2\times 2^{1/3}-3\times 6^{2/3}-3^{1/3}\,x}{104976\,\left(9\times 2^{1/3}-4\times 3^{1/3}\right)\left(6+3\times 2^{2/3}\times 3^{1/3}\,x+x^2\right)}+$$

$$\frac{\text{ArcTan}\left[\frac{3\,(-3)^{1/3}\,2^{2/3}-2\,x}{\sqrt{6\,\left(4-3\,(-3)^{2/3}\,2^{1/3}\right)}}\right]}{26244\,\sqrt{3}\,\left(8-9\,i\,2^{1/3}\times 3^{1/6}+3\times 2^{1/3}\times 3^{2/3}\right)^{3/2}}-$$

$$\left(\left(9\,i-3^{1/3}\left(2\,i\,2^{2/3}+9\times 3^{1/6}+2\times 2^{2/3}\,\sqrt{3}\right)\right)\text{ArcTan}\left[\frac{3\,(-3)^{1/3}\,2^{2/3}-2\,x}{\sqrt{6\,\left(4-3\,(-3)^{2/3}\,2^{1/3}\right)}}\right]\right)\Big/$$

$$\left(209952\,\left(1+(-1)^{1/3}\right)^5\sqrt{2\,\left(4-3\,(-3)^{2/3}\,2^{1/3}\right)}\right)-$$

$$\frac{\text{ArcTan}\left[\frac{3\,(-2)^{2/3}\,3^{1/3}+2\,x}{\sqrt{6\,\left(4+3\,(-2)^{1/3}\,3^{2/3}\right)}}\right]}{26244\,\sqrt{3}\,\left(8+9\,i\,2^{1/3}\times 3^{1/6}+3\times 2^{1/3}\times 3^{2/3}\right)^{3/2}}+$$

$$\frac{\left(9\,i+3^{1/3}\left(4\,i\,2^{2/3}-9\times 3^{1/6}\right)\right)\text{ArcTan}\left[\frac{3\,(-2)^{2/3}\,3^{1/3}+2\,x}{\sqrt{6\,\left(4+3\,(-2)^{1/3}\,3^{2/3}\right)}}\right]}{209952\,\left(1+(-1)^{1/3}\right)^5\sqrt{2\,\left(4+3\,(-2)^{1/3}\,3^{2/3}\right)}}-\frac{\text{ArcTanh}\left[\frac{2^{1/6}\left(3\times 3^{1/3}+2^{1/3}\,x\right)}{\sqrt{3\,\left(-4+3\times 2^{1/3}\times 3^{2/3}\right)}}\right]}{52488\,\sqrt{6}\,\left(-4+3\times 2^{1/3}\times 3^{2/3}\right)^{3/2}}+$$

$$\frac{\left(2\times 2^{2/3}-3\times 3^{2/3}\right)\text{ArcTanh}\left[\frac{2^{1/6}\left(3\times 3^{1/3}+2^{1/3}\,x\right)}{\sqrt{3\,\left(-4+3\times 2^{1/3}\times 3^{2/3}\right)}}\right]}{944784\times 3^{1/6}\,\sqrt{2\,\left(-4+3\times 2^{1/3}\times 3^{2/3}\right)}}-\frac{i\,\text{Log}\left[6-3\,(-3)^{1/3}\,2^{2/3}\,x+x^2\right]}{23328\times 2^{2/3}\times 3^{5/6}\left(1+(-1)^{1/3}\right)^5}+$$

$$\frac{\left(i+\sqrt{3}\right)\text{Log}\left[6+3\,(-2)^{2/3}\,3^{1/3}\,x+x^2\right]}{46656\times 2^{2/3}\times 3^{5/6}\left(1+(-1)^{1/3}\right)^5}+\frac{\text{Log}\left[6+3\times 2^{2/3}\times 3^{1/3}\,x+x^2\right]}{629856\times 2^{2/3}\times 3^{1/3}}$$

Result (type 7, 167 leaves) :

$$\frac{972 - 3942\,x + 648\,x^2 + 96\,x^3 - 27\,x^4 + 4\,x^5}{3\,691\,656\,\left(216 + 108\,x^2 + 324\,x^3 + 18\,x^4 + x^6\right)} +$$
$$\frac{1}{11\,074\,968}\text{RootSum}\Big[216 + 108\,\#1^2 + 324\,\#1^3 + 18\,\#1^4 + \#1^6\,\&,$$
$$\left(1971\,\text{Log}[x - \#1] - 162\,\text{Log}[x - \#1]\,\#1 + 72\,\text{Log}[x - \#1]\,\#1^2 - 27\,\text{Log}[x - \#1]\,\#1^3 + 2\,\text{Log}[x - \#1]\,\#1^4\right) \Big/ \left(36\,\#1 + 162\,\#1^2 + 12\,\#1^3 + \#1^5\right)\,\&\Big]$$

Problem 157: Result is not expressed in closed-form.

$$\int \frac{x^2}{\left(216 + 108\,x^2 + 324\,x^3 + 18\,x^4 + x^6\right)^2}\,dx$$

Optimal (type 3, 986 leaves, 23 steps):

$$-\left(\left(27\left((-2)^{2/3}+2(-1)^{1/3}3^{2/3}\right)-6^{1/3}\left(9+(-3)^{1/3}2^{2/3}\right)x\right)\Big/\right.$$
$$\left.\left(104976\times 2^{2/3}\left(1+(-1)^{1/3}\right)^4\left(4-3(-3)^{2/3}2^{1/3}\right)\left(6-3(-3)^{1/3}2^{2/3}x+x^2\right)\right)\right)-$$
$$\left(27\times 2^{2/3}\left(1+(-2)^{1/3}3^{2/3}\right)-(-1)^{1/3}3^{2/3}\left(2+3(-2)^{1/3}3^{2/3}\right)x\right)\Big/$$
$$\left(472392\times 2^{2/3}\left(8+9\,\mathrm{i}\,2^{1/3}\times 3^{1/6}+3\times 2^{1/3}\times 3^{2/3}\right)\left(6+3(-2)^{2/3}3^{1/3}x+x^2\right)\right)+$$
$$\frac{9\left(6-2^{2/3}\times 3^{1/3}\right)-\left(2-3\times 2^{1/3}\times 3^{2/3}\right)x}{314928\times 2^{2/3}\times 3^{1/3}\left(4-3\times 2^{1/3}\times 3^{2/3}\right)\left(6+3\times 2^{2/3}\times 3^{1/3}x+x^2\right)}-$$
$$\frac{\left(1+\mathrm{i}\sqrt{3}+3\times 2^{1/3}\times 3^{2/3}\right)\mathrm{ArcTan}\left[\frac{3(-3)^{1/3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}2^{1/3}\right)}}\right]}{8748\times 2^{2/3}\times 3^{5/6}\left(1+(-1)^{1/3}\right)^4\left(8-9\,\mathrm{i}\,2^{1/3}\times 3^{1/6}+3\times 2^{1/3}\times 3^{2/3}\right)^{3/2}}+$$
$$\frac{\left(3(-3)^{2/3}+(-1)^{1/3}2^{2/3}\right)\mathrm{ArcTan}\left[\frac{3(-2)^{2/3}3^{1/3}+2x}{\sqrt{6\left(4+3(-2)^{1/3}3^{2/3}\right)}}\right]}{17496\times 6^{5/6}\left(1-(-1)^{1/3}\right)^2\left(1+(-1)^{1/3}\right)^4\left(4+3(-2)^{1/3}3^{2/3}\right)^{3/2}}+$$
$$\frac{\left(\mathrm{i}+\sqrt{3}\right)\mathrm{ArcTan}\left[\frac{3(-2)^{2/3}3^{1/3}+2x}{\sqrt{6\left(4+3(-2)^{1/3}3^{2/3}\right)}}\right]}{34992\times 2^{1/6}\times 3^{1/3}\left(1+(-1)^{1/3}\right)^5\sqrt{4+3(-2)^{1/3}3^{2/3}}}+$$
$$\frac{\mathrm{i}\,\mathrm{ArcTan}\left[\frac{2^{1/6}\left(3(-3)^{1/3}-2^{1/3}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}2^{1/3}\right)}}\right]}{17496\times 2^{1/6}\times 3^{1/3}\left(1+(-1)^{1/3}\right)^5\sqrt{4-3(-3)^{2/3}2^{1/3}}}-$$
$$\frac{\left(2^{2/3}-3\times 3^{2/3}\right)\mathrm{ArcTanh}\left[\frac{2^{1/6}\left(3\times 3^{1/3}+2^{1/3}x\right)}{\sqrt{3\left(-4+3\times 2^{1/3}\times 3^{2/3}\right)}}\right]}{17496\times 6^{5/6}\left(1-(-1)^{1/3}\right)^2\left(1+(-1)^{1/3}\right)^4\left(-4+3\times 2^{1/3}\times 3^{2/3}\right)^{3/2}}-$$
$$\frac{\mathrm{ArcTanh}\left[\frac{2^{1/6}\left(3\times 3^{1/3}+2^{1/3}x\right)}{\sqrt{3\left(-4+3\times 2^{1/3}\times 3^{2/3}\right)}}\right]}{157464\times 2^{1/6}\times 3^{5/6}\sqrt{-4+3\times 2^{1/3}\times 3^{2/3}}}+\frac{\left(\mathrm{i}+\sqrt{3}\right)\mathrm{Log}\left[6-3(-3)^{1/3}2^{2/3}x+x^2\right]}{419904\times 2^{1/3}\times 3^{1/6}\left(1+(-1)^{1/3}\right)^5}-$$
$$\frac{\mathrm{i}\,\mathrm{Log}\left[6+3(-2)^{2/3}3^{1/3}x+x^2\right]}{209952\times 2^{1/3}\times 3^{1/6}\left(1+(-1)^{1/3}\right)^5}+\frac{\mathrm{Log}\left[6+3\times 2^{2/3}\times 3^{1/3}x+x^2\right]}{1889568\times 2^{1/3}\times 3^{2/3}}$$

Result (type 7, 167 leaves):

$$\frac{-7884+324 x-2724 x^2-216 x^3+8 x^4-9 x^5}{7383312\left(216+108 x^2+324 x^3+18 x^4+x^6\right)}-$$
$$\frac{1}{44299872}\text{RootSum}\Big[216+108 \#1^2+324 \#1^3+18 \#1^4+\#1^6\&,$$
$$\Big(324 \text{Log}[x-\#1]+2436 \text{Log}[x-\#1] \#1+324 \text{Log}[x-\#1] \#1^2-$$
$$16 \text{Log}[x-\#1] \#1^3+9 \text{Log}[x-\#1] \#1^4\Big) \Big/ \left(36 \#1+162 \#1^2+12 \#1^3+\#1^5\right)\&\Big]$$

Problem 160: Result more than twice size of optimal antiderivative.

$$\int (b+2 c x) \left(b x+c x^2\right)^{13} dx$$

Optimal (type 1, 15 leaves, 1 step):

$$\frac{1}{14}\left(b x+c x^2\right)^{14}$$

Result (type 1, 172 leaves):

$$\frac{b^{14} x^{14}}{14}+b^{13} c x^{15}+\frac{13}{2} b^{12} c^2 x^{16}+26 b^{11} c^3 x^{17}+\frac{143}{2} b^{10} c^4 x^{18}+$$
$$143 b^9 c^5 x^{19}+\frac{429}{2} b^8 c^6 x^{20}+\frac{1716}{7} b^7 c^7 x^{21}+\frac{429}{2} b^6 c^8 x^{22}+143 b^5 c^9 x^{23}+$$
$$\frac{143}{2} b^4 c^{10} x^{24}+26 b^3 c^{11} x^{25}+\frac{13}{2} b^2 c^{12} x^{26}+b c^{13} x^{27}+\frac{c^{14} x^{28}}{14}$$

Problem 161: Result more than twice size of optimal antiderivative.

$$\int x^{14} \left(b+2 c x^2\right) \left(b x+c x^3\right)^{13} dx$$

Optimal (type 1, 16 leaves, 3 steps):

$$\frac{1}{28} x^{28}\left(b+c x^2\right)^{14}$$

Result (type 1, 182 leaves):

$$\frac{b^{14} x^{28}}{28}+\frac{1}{2} b^{13} c x^{30}+\frac{13}{4} b^{12} c^2 x^{32}+13 b^{11} c^3 x^{34}+\frac{143}{4} b^{10} c^4 x^{36}+$$
$$\frac{143}{2} b^9 c^5 x^{38}+\frac{429}{4} b^8 c^6 x^{40}+\frac{858}{7} b^7 c^7 x^{42}+\frac{429}{4} b^6 c^8 x^{44}+\frac{143}{2} b^5 c^9 x^{46}+$$
$$\frac{143}{4} b^4 c^{10} x^{48}+13 b^3 c^{11} x^{50}+\frac{13}{4} b^2 c^{12} x^{52}+\frac{1}{2} b c^{13} x^{54}+\frac{c^{14} x^{56}}{28}$$

Problem 162: Result more than twice size of optimal antiderivative.

$$\int x^{28} \left(b+2 c x^3\right) \left(b x+c x^4\right)^{13} dx$$

Optimal (type 1, 16 leaves, 3 steps):

$$\frac{1}{42} x^{42} \left(b + c x^3\right)^{14}$$

Result (type 1, 186 leaves):

$$\frac{b^{14} x^{42}}{42} + \frac{1}{3} b^{13} c x^{45} + \frac{13}{6} b^{12} c^2 x^{48} + \frac{26}{3} b^{11} c^3 x^{51} + \frac{143}{6} b^{10} c^4 x^{54} + \frac{143}{3} b^9 c^5 x^{57} + \frac{143}{2} b^8 c^6 x^{60} + \frac{572}{7} b^7 c^7 x^{63} + \frac{143}{2} b^6 c^8 x^{66} + \frac{143}{3} b^5 c^9 x^{69} + \frac{143}{6} b^4 c^{10} x^{72} + \frac{26}{3} b^3 c^{11} x^{75} + \frac{13}{6} b^2 c^{12} x^{78} + \frac{1}{3} b c^{13} x^{81} + \frac{c^{14} x^{84}}{42}$$

Problem 171: Result more than twice size of optimal antiderivative.

$$\int \frac{x^{-7 (-1+n)} \left(b + 2 c x^n\right)}{\left(b x + c x^{1+n}\right)^8} \, dx$$

Optimal (type 3, 21 leaves, 3 steps):

$$-\frac{x^{-7 n}}{7 n \left(b + c x^n\right)^7}$$

Result (type 3, 127 leaves):

$$-\frac{1}{7 b^{14} n \left(b + c x^n\right)^7} x^{-7 n} \left(b^{14} + 1716 b^7 c^7 x^{7 n} + 12012 b^6 c^8 x^{8 n} + 36036 b^5 c^9 x^{9 n} + 60060 b^4 c^{10} x^{10 n} + 60060 b^3 c^{11} x^{11 n} + 36036 b^2 c^{12} x^{12 n} + 12012 b c^{13} x^{13 n} + 1716 c^{14} x^{14 n}\right)$$

Problem 192: Result more than twice size of optimal antiderivative.

$$\int \left(b + 2 c x + 3 d x^2\right) \left(a + b x + c x^2 + d x^3\right)^7 \, dx$$

Optimal (type 1, 21 leaves, 1 step):

$$\frac{1}{8} \left(a + b x + c x^2 + d x^3\right)^8$$

Result (type 1, 143 leaves):

$$\frac{1}{8} x \left(b + x \left(c + d x\right)\right) \left(8 a^7 + 28 a^6 x \left(b + x \left(c + d x\right)\right) + 56 a^5 x^2 \left(b + x \left(c + d x\right)\right)^2 + 70 a^4 x^3 \left(b + x \left(c + d x\right)\right)^3 + 56 a^3 x^4 \left(b + x \left(c + d x\right)\right)^4 + 28 a^2 x^5 \left(b + x \left(c + d x\right)\right)^5 + 8 a x^6 \left(b + x \left(c + d x\right)\right)^6 + x^7 \left(b + x \left(c + d x\right)\right)^7\right)$$

Problem 195: Result more than twice size of optimal antiderivative.

$$\int \left(b + 3 d x^2\right) \left(a + b x + d x^3\right)^7 \, dx$$

Optimal (type 1, 16 leaves, 1 step):

$$\frac{1}{8}\left(a+b x+d x^3\right)^8$$

Result (type 1, 127 leaves):

$$\frac{1}{8} x \left(b+d x^2\right) \left(8 a^7+28 a^6 x \left(b+d x^2\right)+56 a^5 x^2 \left(b+d x^2\right)^2+70 a^4 x^3 \left(b+d x^2\right)^3+56 a^3 x^4 \left(b+d x^2\right)^4+28 a^2 x^5 \left(b+d x^2\right)^5+8 a x^6 \left(b+d x^2\right)^6+x^7 \left(b+d x^2\right)^7\right)$$

Problem 196: Result more than twice size of optimal antiderivative.

$$\int \left(b+3 d x^2\right) \left(b x+d x^3\right)^7 \, dx$$

Optimal (type 1, 15 leaves, 1 step):

$$\frac{1}{8}\left(b x+d x^3\right)^8$$

Result (type 1, 98 leaves):

$$\frac{b^8 x^8}{8}+b^7 d x^{10}+\frac{7}{2} b^6 d^2 x^{12}+7 b^5 d^3 x^{14}+\frac{35}{4} b^4 d^4 x^{16}+7 b^3 d^5 x^{18}+\frac{7}{2} b^2 d^6 x^{20}+b d^7 x^{22}+\frac{d^8 x^{24}}{8}$$

Problem 197: Result more than twice size of optimal antiderivative.

$$\int x^7 \left(b+d x^2\right)^7 \left(b+3 d x^2\right) \, dx$$

Optimal (type 1, 16 leaves, 2 steps):

$$\frac{1}{8} x^8 \left(b+d x^2\right)^8$$

Result (type 1, 98 leaves):

$$\frac{b^8 x^8}{8}+b^7 d x^{10}+\frac{7}{2} b^6 d^2 x^{12}+7 b^5 d^3 x^{14}+\frac{35}{4} b^4 d^4 x^{16}+7 b^3 d^5 x^{18}+\frac{7}{2} b^2 d^6 x^{20}+b d^7 x^{22}+\frac{d^8 x^{24}}{8}$$

Problem 198: Result more than twice size of optimal antiderivative.

$$\int \left(2 c x+3 d x^2\right) \left(a+c x^2+d x^3\right)^7 \, dx$$

Optimal (type 1, 18 leaves, 1 step):

$$\frac{1}{8}\left(a+c x^2+d x^3\right)^8$$

Result (type 1, 115 leaves):

$$\frac{1}{8} x^2 (c+d x) \left(8 a^7+28 a^6 x^2 (c+d x)+56 a^5 x^4 (c+d x)^2+70 a^4 x^6 (c+d x)^3+56 a^3 x^8 (c+d x)^4+28 a^2 x^{10} (c+d x)^5+8 a x^{12} (c+d x)^6+x^{14} (c+d x)^7\right)$$

Problem 199: Result more than twice size of optimal antiderivative.

$$\int \left(2 c x+3 d x^2\right) \left(c x^2+d x^3\right)^7 \, dx$$

Optimal (type 1, 17 leaves, 1 step):

$$\frac{1}{8} \left(c x^2+d x^3\right)^8$$

Result (type 1, 98 leaves):

$$\frac{c^8 x^{16}}{8}+c^7 d x^{17}+\frac{7}{2} c^6 d^2 x^{18}+7 c^5 d^3 x^{19}+\frac{35}{4} c^4 d^4 x^{20}+7 c^3 d^5 x^{21}+\frac{7}{2} c^2 d^6 x^{22}+c d^7 x^{23}+\frac{d^8 x^{24}}{8}$$

Problem 200: Result more than twice size of optimal antiderivative.

$$\int x^7 \left(c x+d x^2\right)^7 \left(2 c x+3 d x^2\right) \, dx$$

Optimal (type 1, 14 leaves, 2 steps):

$$\frac{1}{8} x^{16} (c+d x)^8$$

Result (type 1, 98 leaves):

$$\frac{c^8 x^{16}}{8}+c^7 d x^{17}+\frac{7}{2} c^6 d^2 x^{18}+7 c^5 d^3 x^{19}+\frac{35}{4} c^4 d^4 x^{20}+7 c^3 d^5 x^{21}+\frac{7}{2} c^2 d^6 x^{22}+c d^7 x^{23}+\frac{d^8 x^{24}}{8}$$

Problem 201: Result more than twice size of optimal antiderivative.

$$\int x^{14} (c+d x)^7 \left(2 c x+3 d x^2\right) \, dx$$

Optimal (type 1, 14 leaves, 1 step):

$$\frac{1}{8} x^{16} (c+d x)^8$$

Result (type 1, 98 leaves):

$$\frac{c^8 x^{16}}{8}+c^7 d x^{17}+\frac{7}{2} c^6 d^2 x^{18}+7 c^5 d^3 x^{19}+\frac{35}{4} c^4 d^4 x^{20}+7 c^3 d^5 x^{21}+\frac{7}{2} c^2 d^6 x^{22}+c d^7 x^{23}+\frac{d^8 x^{24}}{8}$$

Problem 202: Result more than twice size of optimal antiderivative.

$$\int x (2 c+3 d x) \left(a+c x^2+d x^3\right)^7 \, dx$$

Optimal (type 1, 18 leaves, 1 step):

$$\frac{1}{8}\left(a+c\,x^2+d\,x^3\right)^8$$

Result (type 1, 115 leaves):

$$\frac{1}{8}x^2\,(c+d\,x)\,\left(8\,a^7+28\,a^6\,x^2\,(c+d\,x)+56\,a^5\,x^4\,(c+d\,x)^2+70\,a^4\,x^6\,(c+d\,x)^3+56\,a^3\,x^8\,(c+d\,x)^4+28\,a^2\,x^{10}\,(c+d\,x)^5+8\,a\,x^{12}\,(c+d\,x)^6+x^{14}\,(c+d\,x)^7\right)$$

Problem 203: Result more than twice size of optimal antiderivative.

$$\int x\,(2\,c+3\,d\,x)\,\left(c\,x^2+d\,x^3\right)^7\,dx$$

Optimal (type 1, 14 leaves, 2 steps):

$$\frac{1}{8}x^{16}\,(c+d\,x)^8$$

Result (type 1, 98 leaves):

$$\frac{c^8\,x^{16}}{8}+c^7\,d\,x^{17}+\frac{7}{2}c^6\,d^2\,x^{18}+7\,c^5\,d^3\,x^{19}+\frac{35}{4}c^4\,d^4\,x^{20}+7\,c^3\,d^5\,x^{21}+\frac{7}{2}c^2\,d^6\,x^{22}+c\,d^7\,x^{23}+\frac{d^8\,x^{24}}{8}$$

Problem 204: Result more than twice size of optimal antiderivative.

$$\int x^8\,(2\,c+3\,d\,x)\,\left(c\,x+d\,x^2\right)^7\,dx$$

Optimal (type 1, 18 leaves, 1 step):

$$\frac{1}{8}x^8\,\left(c\,x+d\,x^2\right)^8$$

Result (type 1, 98 leaves):

$$\frac{c^8\,x^{16}}{8}+c^7\,d\,x^{17}+\frac{7}{2}c^6\,d^2\,x^{18}+7\,c^5\,d^3\,x^{19}+\frac{35}{4}c^4\,d^4\,x^{20}+7\,c^3\,d^5\,x^{21}+\frac{7}{2}c^2\,d^6\,x^{22}+c\,d^7\,x^{23}+\frac{d^8\,x^{24}}{8}$$

Problem 205: Result more than twice size of optimal antiderivative.

$$\int x^{15}\,(c+d\,x)^7\,(2\,c+3\,d\,x)\,dx$$

Optimal (type 1, 14 leaves, 1 step):

$$\frac{1}{8}x^{16}\,(c+d\,x)^8$$

Result (type 1, 98 leaves):

$$\frac{c^8\,x^{16}}{8}+c^7\,d\,x^{17}+\frac{7}{2}c^6\,d^2\,x^{18}+7\,c^5\,d^3\,x^{19}+\frac{35}{4}c^4\,d^4\,x^{20}+7\,c^3\,d^5\,x^{21}+\frac{7}{2}c^2\,d^6\,x^{22}+c\,d^7\,x^{23}+\frac{d^8\,x^{24}}{8}$$

Problem 206: Result more than twice size of optimal antiderivative.

$$\int (a+b x) \left(1+\left(a x+\frac{b x^2}{2}\right)^4\right) dx$$

Optimal (type 1, 28 leaves, 2 steps):

$$a x+\frac{b x^2}{2}+\frac{1}{160} x^5 (2 a+b x)^5$$

Result (type 1, 80 leaves):

$$a x+\frac{b x^2}{2}+\frac{a^5 x^5}{5}+\frac{1}{2} a^4 b x^6+\frac{1}{2} a^3 b^2 x^7+\frac{1}{4} a^2 b^3 x^8+\frac{1}{16} a b^4 x^9+\frac{b^5 x^{10}}{160}$$

Problem 207: Result more than twice size of optimal antiderivative.

$$\int (a+b x) \left(1+\left(c+a x+\frac{b x^2}{2}\right)^4\right) dx$$

Optimal (type 1, 31 leaves, 2 steps):

$$a x+\frac{b x^2}{2}+\frac{1}{5} \left(c+a x+\frac{b x^2}{2}\right)^5$$

Result (type 1, 108 leaves):

$$\frac{1}{160} x (2 a+b x) \left(80+80 c^4+16 a^4 x^4+32 a^3 b x^5+24 a^2 b^2 x^6+8 a b^3 x^7+b^4 x^8+80 c^3 x (2 a+b x)+40 c^2 x^2 (2 a+b x)^2+10 c x^3 (2 a+b x)^3\right)$$

Problem 209: Result more than twice size of optimal antiderivative.

$$\int (a+b x) \left(1+\left(c+a x+\frac{b x^2}{2}\right)^n\right) dx$$

Optimal (type 3, 35 leaves, 2 steps):

$$a x+\frac{b x^2}{2}+\frac{\left(c+a x+\frac{b x^2}{2}\right)^{1+n}}{1+n}$$

Result (type 3, 73 leaves):

$$\frac{1}{2 (1+n)}\left(2 c \left(c+a x+\frac{b x^2}{2}\right)^n+2 a x \left(1+n+\left(c+a x+\frac{b x^2}{2}\right)^n\right)+b x^2 \left(1+n+\left(c+a x+\frac{b x^2}{2}\right)^n\right)\right)$$

Problem 210: Result more than twice size of optimal antiderivative.

$$\int \left(a+c x^2\right) \left(1+\left(a x+\frac{c x^3}{3}\right)^5\right) dx$$

Optimal (type 1, 30 leaves, 2 steps):

$$a x+\frac{c x^3}{3}+\frac{1}{6}\left(a x+\frac{c x^3}{3}\right)^6$$

Result (type 1, 93 leaves):

$$a x+\frac{c x^3}{3}+\frac{a^6 x^6}{6}+\frac{1}{3} a^5 c x^8+\frac{5}{18} a^4 c^2 x^{10}+\frac{10}{81} a^3 c^3 x^{12}+\frac{5}{162} a^2 c^4 x^{14}+\frac{1}{243} a c^5 x^{16}+\frac{c^6 x^{18}}{4374}$$

Problem 211: Result more than twice size of optimal antiderivative.

$$\int\left(a+c x^2\right)\left(1+\left(d+a x+\frac{c x^3}{3}\right)^5\right)dx$$

Optimal (type 1, 31 leaves, 2 steps):

$$a x+\frac{c x^3}{3}+\frac{1}{6}\left(d+a x+\frac{c x^3}{3}\right)^6$$

Result (type 1, 140 leaves):

$$\frac{1}{4374} x\left(3 a+c x^2\right)\left(1458+1458 d^5+243 a^5 x^5+405 a^4 c x^7+270 a^3 c^2 x^9+90 a^2 c^3 x^{11}+15 a c^4 x^{13}+c^5 x^{15}+1215 d^4\left(3 a x+c x^3\right)+540 d^3\left(3 a x+c x^3\right)^2+135 d^2\left(3 a x+c x^3\right)^3+18 d\left(3 a x+c x^3\right)^4\right)$$

Problem 212: Result more than twice size of optimal antiderivative.

$$\int\left(b x+c x^2\right)\left(1+\left(\frac{b x^2}{2}+\frac{c x^3}{3}\right)^5\right)dx$$

Optimal (type 1, 34 leaves, 2 steps):

$$\frac{b x^2}{2}+\frac{c x^3}{3}+\frac{x^{12}\left(3 b+2 c x\right)^6}{279936}$$

Result (type 1, 98 leaves):

$$\frac{b x^2}{2}+\frac{c x^3}{3}+\frac{b^6 x^{12}}{384}+\frac{1}{96} b^5 c x^{13}+\frac{5}{288} b^4 c^2 x^{14}+\frac{5}{324} b^3 c^3 x^{15}+\frac{5}{648} b^2 c^4 x^{16}+\frac{1}{486} b c^5 x^{17}+\frac{c^6 x^{18}}{4374}$$

Problem 213: Result more than twice size of optimal antiderivative.

$$\int\left(b x+c x^2\right)\left(1+\left(d+\frac{b x^2}{2}+\frac{c x^3}{3}\right)^5\right)dx$$

Optimal (type 1, 41 leaves, 2 steps):

$$\frac{b x^2}{2}+\frac{c x^3}{3}+\frac{1}{6}\left(d+\frac{b x^2}{2}+\frac{c x^3}{3}\right)^6$$

Result (type 1, 146 leaves):

$$\frac{1}{279936} x^2 (3 b + 2 c x) \left(46656 + 46656 d^5 + 243 b^5 x^{10} + 810 b^4 c x^{11} + 1080 b^3 c^2 x^{12} + 720 b^2 c^3 x^{13} + 240 b c^4 x^{14} + 32 c^5 x^{15} + 19440 d^4 x^2 (3 b + 2 c x) + 4320 d^3 x^4 (3 b + 2 c x)^2 + 540 d^2 x^6 (3 b + 2 c x)^3 + 36 d x^8 (3 b + 2 c x)^4\right)$$

Problem 214: Result more than twice size of optimal antiderivative.

$$\int \left(a + b x + c x^2\right) \left(1 + \left(a x + \frac{b x^2}{2} + \frac{c x^3}{3}\right)^5\right) dx$$

Optimal (type 1, 46 leaves, 2 steps):

$$a x + \frac{b x^2}{2} + \frac{c x^3}{3} + \frac{1}{6} \left(a x + \frac{b x^2}{2} + \frac{c x^3}{3}\right)^6$$

Result (type 1, 244 leaves):

$$\frac{a^6 x^6}{6} + \frac{1}{6} a^5 x^7 (3 b + 2 c x) + \frac{5}{72} a^4 x^8 (3 b + 2 c x)^2 + \frac{5}{324} a^3 x^9 (3 b + 2 c x)^3 + \frac{5 a^2 x^{10} (3 b + 2 c x)^4}{2592} + a \left(x + \frac{b^5 x^{11}}{32} + \frac{5}{48} b^4 c x^{12} + \frac{5}{36} b^3 c^2 x^{13} + \frac{5}{54} b^2 c^3 x^{14} + \frac{5}{162} b c^4 x^{15} + \frac{c^5 x^{16}}{243}\right) + \frac{1}{279936} x^2 \left(729 b^6 x^{10} + 2916 b^5 c x^{11} + 4860 b^4 c^2 x^{12} + 4320 b^3 c^3 x^{13} + 2160 b^2 c^4 x^{14} + 576 b \left(243 + c^5 x^{15}\right) + 64 c x \left(1458 + c^5 x^{15}\right)\right)$$

Problem 215: Result more than twice size of optimal antiderivative.

$$\int \left(a + b x + c x^2\right) \left(1 + \left(d + a x + \frac{b x^2}{2} + \frac{c x^3}{3}\right)^5\right) dx$$

Optimal (type 1, 47 leaves, 2 steps):

$$a x + \frac{b x^2}{2} + \frac{c x^3}{3} + \frac{1}{6} \left(d + a x + \frac{b x^2}{2} + \frac{c x^3}{3}\right)^6$$

Result (type 1, 248 leaves):

$$\frac{1}{279936} x (6 a + x (3 b + 2 c x)) \left(46656 + 46656 d^5 + 7776 a^5 x^5 + 243 b^5 x^{10} + 810 b^4 c x^{11} + 1080 b^3 c^2 x^{12} + 720 b^2 c^3 x^{13} + 240 b c^4 x^{14} + 32 c^5 x^{15} + 6480 a^4 x^6 (3 b + 2 c x) + 2160 a^3 x^7 (3 b + 2 c x)^2 + 360 a^2 x^8 (3 b + 2 c x)^3 + 30 a x^9 (3 b + 2 c x)^4 + 19440 d^4 x (6 a + x (3 b + 2 c x)) + 4320 d^3 x^2 (6 a + x (3 b + 2 c x))^2 + 540 d^2 x^3 (6 a + x (3 b + 2 c x))^3 + 36 d x^4 (6 a + x (3 b + 2 c x))^4\right)$$

Problem 221: Result more than twice size of optimal antiderivative.

$$\int (1+2x)\left(x+x^2\right)^3\left(-18+7\left(x+x^2\right)^3\right)^2\,dx$$

Optimal (type 1, 33 leaves, ? steps):

$$81x^4(1+x)^4-36x^7(1+x)^7+\frac{49}{10}x^{10}(1+x)^{10}$$

Result (type 1, 96 leaves):

$$81x^4+324x^5+486x^6+288x^7-171x^8-756x^9-\frac{12551x^{10}}{10}-1211x^{11}-\frac{1071x^{12}}{2}+336x^{13}+993x^{14}+\frac{6174x^{15}}{5}+1029x^{16}+588x^{17}+\frac{441x^{18}}{2}+49x^{19}+\frac{49x^{20}}{10}$$

Problem 222: Result more than twice size of optimal antiderivative.

$$\int x^3(1+x)^3(1+2x)\left(-18+7x^3(1+x)^3\right)^2\,dx$$

Optimal (type 1, 33 leaves, ? steps):

$$81x^4(1+x)^4-36x^7(1+x)^7+\frac{49}{10}x^{10}(1+x)^{10}$$

Result (type 1, 96 leaves):

$$81x^4+324x^5+486x^6+288x^7-171x^8-756x^9-\frac{12551x^{10}}{10}-1211x^{11}-\frac{1071x^{12}}{2}+336x^{13}+993x^{14}+\frac{6174x^{15}}{5}+1029x^{16}+588x^{17}+\frac{441x^{18}}{2}+49x^{19}+\frac{49x^{20}}{10}$$

Problem 227: Result is not expressed in closed-form.

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx+cx^2+bx^3+ax^4}\,dx$$

Optimal (type 3, 605 leaves, 9 steps):

$$\left(\left(4 a^2 B + b \left(b - \sqrt{8 a^2 + b^2 - 4 a c}\right) D - a \left(A \left(b - \sqrt{8 a^2 + b^2 - 4 a c}\right) + b C - \sqrt{8 a^2 + b^2 - 4 a c}\, C + 2 c D\right)\right)\right.$$

$$\left.\text{ArcTan}\left[\frac{b - \sqrt{8 a^2 + b^2 - 4 a c} + 4 a x}{\sqrt{2} \sqrt{4 a^2 + 2 a c - b \left(b - \sqrt{8 a^2 + b^2 - 4 a c}\right)}}\right]\right) \Big/$$

$$\left(\sqrt{2}\, a \sqrt{8 a^2 + b^2 - 4 a c} \sqrt{4 a^2 + 2 a c - b \left(b - \sqrt{8 a^2 + b^2 - 4 a c}\right)}\right) -$$

$$\left(\left(4 a^2 B + b \left(b + \sqrt{8 a^2 + b^2 - 4 a c}\right) D -\right.\right.$$

$$\left.a \left(A \left(b + \sqrt{8 a^2 + b^2 - 4 a c}\right) + b C + \sqrt{8 a^2 + b^2 - 4 a c}\, C + 2 c D\right)\right)$$

$$\left.\text{ArcTan}\left[\frac{b + \sqrt{8 a^2 + b^2 - 4 a c} + 4 a x}{\sqrt{2} \sqrt{4 a^2 + 2 a c - b \left(b + \sqrt{8 a^2 + b^2 - 4 a c}\right)}}\right]\right) \Big/$$

$$\left(\sqrt{2}\, a \sqrt{8 a^2 + b^2 - 4 a c} \sqrt{4 a^2 + 2 a c - b \left(b + \sqrt{8 a^2 + b^2 - 4 a c}\right)}\right) -$$

$$\left(\left(2 a (A - C) + \left(b - \sqrt{8 a^2 + b^2 - 4 a c}\right) D\right) \text{Log}\left[2 a + \left(b - \sqrt{8 a^2 + b^2 - 4 a c}\right) x + 2 a x^2\right]\right) \Big/$$

$$\left(4 a \sqrt{8 a^2 + b^2 - 4 a c}\right) +$$

$$\left(\left(2 a (A - C) + \left(b + \sqrt{8 a^2 + b^2 - 4 a c}\right) D\right) \text{Log}\left[2 a + \left(b + \sqrt{8 a^2 + b^2 - 4 a c}\right) x + 2 a x^2\right]\right) \Big/$$

$$\left(4 a \sqrt{8 a^2 + b^2 - 4 a c}\right)$$

Result (type 7, 98 leaves) :

$$\text{RootSum}\left[a + b\, \#1 + c\, \#1^2 + b\, \#1^3 + a\, \#1^4\, \&,\right.$$
$$\left(A\, \text{Log}[x - \#1] + B\, \text{Log}[x - \#1]\, \#1 + C\, \text{Log}[x - \#1]\, \#1^2 + D\, \text{Log}[x - \#1]\, \#1^3\right) \Big/$$
$$\left.\left(b + 2 c\, \#1 + 3 b\, \#1^2 + 4 a\, \#1^3\right)\, \&\right]$$

Problem 250: Result is not expressed in closed-form.

$$\int \frac{x^3 \left(5 + x + 3 x^2 + 2 x^3\right)}{2 + x + 5 x^2 + x^3 + 2 x^4}\, dx$$

Optimal (type 3, 307 leaves, 13 steps):

$$-\frac{1}{28}\left(35-9 i \sqrt{7}\right) x-\frac{1}{28}\left(35+9 i \sqrt{7}\right) x+\frac{1}{28}\left(7-5 i \sqrt{7}\right) x^2+\frac{1}{28}\left(7+5 i \sqrt{7}\right) x^2+\frac{1}{42}\left(7-5 i \sqrt{7}\right) x^3+\frac{1}{42}\left(7+5 i \sqrt{7}\right) x^3+\frac{11\left(9 i+5 \sqrt{7}\right) \text{ArcTan}\left[\frac{1-i \sqrt{7}+8 x}{\sqrt{2\left(35+i \sqrt{7}\right)}}\right]}{4 \sqrt{14\left(35+i \sqrt{7}\right)}}-\frac{11\left(9 i-5 \sqrt{7}\right) \text{ArcTan}\left[\frac{1+i \sqrt{7}+8 x}{\sqrt{2\left(35-i \sqrt{7}\right)}}\right]}{4 \sqrt{14\left(35-i \sqrt{7}\right)}}+\frac{3}{112}\left(7-11 i \sqrt{7}\right) \text{Log}\left[4+\left(1-i \sqrt{7}\right) x+4 x^2\right]+\frac{3}{112}\left(7+11 i \sqrt{7}\right) \text{Log}\left[4+\left(1+i \sqrt{7}\right) x+4 x^2\right]$$

Result (type 7, 109 leaves):

$$\frac{1}{6}\left(x\left(-15+3 x+2 x^2\right)+3\,\text{RootSum}\left[2+\#1+5\,\#1^2+\#1^3+2\,\#1^4\,\&,\left(10\,\text{Log}[x-\#1]+\text{Log}[x-\#1]\,\#1+19\,\text{Log}[x-\#1]\,\#1^2+3\,\text{Log}[x-\#1]\,\#1^3\right)\Big/\left(1+10\,\#1+3\,\#1^2+8\,\#1^3\right)\,\&\right]\right)$$

Problem 251: Result is not expressed in closed-form.

$$\int \frac{x^2\left(5+x+3 x^2+2 x^3\right)}{2+x+5 x^2+x^3+2 x^4}\,dx$$

Optimal (type 3, 269 leaves, 13 steps):

$$\frac{1}{14}\left(7-5 i \sqrt{7}\right) x+\frac{1}{14}\left(7+5 i \sqrt{7}\right) x+\frac{1}{28}\left(7-5 i \sqrt{7}\right) x^2+\frac{1}{28}\left(7+5 i \sqrt{7}\right) x^2-\frac{\left(53 i+\sqrt{7}\right) \text{ArcTan}\left[\frac{1-i \sqrt{7}+8 x}{\sqrt{2\left(35+i \sqrt{7}\right)}}\right]}{2 \sqrt{14\left(35+i \sqrt{7}\right)}}+\frac{\left(53 i-\sqrt{7}\right) \text{ArcTan}\left[\frac{1+i \sqrt{7}+8 x}{\sqrt{2\left(35-i \sqrt{7}\right)}}\right]}{2 \sqrt{14\left(35-i \sqrt{7}\right)}}-\frac{1}{56}\left(35+9 i \sqrt{7}\right) \text{Log}\left[4+\left(1-i \sqrt{7}\right) x+4 x^2\right]-\frac{1}{56}\left(35-9 i \sqrt{7}\right) \text{Log}\left[4+\left(1+i \sqrt{7}\right) x+4 x^2\right]$$

Result (type 7, 101 leaves):

$$x+\frac{x^2}{2}-\text{RootSum}\left[2+\#1+5\,\#1^2+\#1^3+2\,\#1^4\,\&,\left(2\,\text{Log}[x-\#1]+3\,\text{Log}[x-\#1]\,\#1+\text{Log}[x-\#1]\,\#1^2+5\,\text{Log}[x-\#1]\,\#1^3\right)\Big/\left(1+10\,\#1+3\,\#1^2+8\,\#1^3\right)\,\&\right]$$

Problem 252: Result is not expressed in closed-form.

$$\int \frac{x\left(5+x+3x^2+2x^3\right)}{2+x+5x^2+x^3+2x^4}\,dx$$

Optimal (type 3, 230 leaves, 11 steps):

$$\frac{1}{14}\left(7-5\,i\,\sqrt{7}\right)x+\frac{1}{14}\left(7+5\,i\,\sqrt{7}\right)x-\frac{\left(19\,i+7\sqrt{7}\right)\text{ArcTan}\left[\frac{1-i\sqrt{7}+8x}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right]}{\sqrt{14\left(35+i\sqrt{7}\right)}}+\frac{\left(19\,i-7\sqrt{7}\right)\text{ArcTan}\left[\frac{1+i\sqrt{7}+8x}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right]}{\sqrt{14\left(35-i\sqrt{7}\right)}}+\frac{1}{28}\left(7+5\,i\,\sqrt{7}\right)\text{Log}\left[4+\left(1-i\sqrt{7}\right)x+4x^2\right]+\frac{1}{28}\left(7-5\,i\,\sqrt{7}\right)\text{Log}\left[4+\left(1+i\sqrt{7}\right)x+4x^2\right]$$

Result (type 7, 94 leaves):

$$x+2\,\text{RootSum}\Big[2+\#1+5\,\#1^2+\#1^3+2\,\#1^4\,\&,\ \left(-\text{Log}[x-\#1]+2\,\text{Log}[x-\#1]\,\#1-2\,\text{Log}[x-\#1]\,\#1^2+\text{Log}[x-\#1]\,\#1^3\right)\Big/\left(1+10\,\#1+3\,\#1^2+8\,\#1^3\right)\,\&\Big]$$

Problem 253: Result is not expressed in closed-form.

$$\int \frac{5+x+3x^2+2x^3}{2+x+5x^2+x^3+2x^4}\,dx$$

Optimal (type 3, 198 leaves, 9 steps):

$$\frac{\left(19\,i+7\sqrt{7}\right)\text{ArcTan}\left[\frac{1-i\sqrt{7}+8x}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right]}{\sqrt{14\left(35+i\sqrt{7}\right)}}-\frac{\left(19\,i-7\sqrt{7}\right)\text{ArcTan}\left[\frac{1+i\sqrt{7}+8x}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right]}{\sqrt{14\left(35-i\sqrt{7}\right)}}+\frac{1}{28}\left(7+5\,i\,\sqrt{7}\right)\text{Log}\left[4+\left(1-i\sqrt{7}\right)x+4x^2\right]+\frac{1}{28}\left(7-5\,i\,\sqrt{7}\right)\text{Log}\left[4+\left(1+i\sqrt{7}\right)x+4x^2\right]$$

Result (type 7, 90 leaves):

$$\text{RootSum}\left[2+\#1+5\,\#1^2+\#1^3+2\,\#1^4\,\&,\ \frac{5\,\text{Log}[x-\#1]+\text{Log}[x-\#1]\,\#1+3\,\text{Log}[x-\#1]\,\#1^2+2\,\text{Log}[x-\#1]\,\#1^3}{1+10\,\#1+3\,\#1^2+8\,\#1^3}\,\&\right]$$

Problem 254: Result is not expressed in closed-form.

$$\int \frac{5+x+3x^2+2x^3}{x\left(2+x+5x^2+x^3+2x^4\right)}\,dx$$

Optimal (type 3, 245 leaves, 13 steps):

$$-\frac{\left(53+i\sqrt{7}\right)\text{ArcTanh}\left[\frac{i-\sqrt{7}+8\,i\,x}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right]}{2\sqrt{14\left(35-i\sqrt{7}\right)}}+\frac{\left(53-i\sqrt{7}\right)\text{ArcTanh}\left[\frac{i+\sqrt{7}+8\,i\,x}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right]}{2\sqrt{14\left(35+i\sqrt{7}\right)}}+$$
$$\frac{1}{28}\left(35-9\,i\sqrt{7}\right)\text{Log}[x]+\frac{1}{28}\left(35+9\,i\sqrt{7}\right)\text{Log}[x]-$$
$$\frac{1}{56}\left(35-9\,i\sqrt{7}\right)\text{Log}\left[4\,i+\left(i-\sqrt{7}\right)x+4\,i\,x^2\right]-\frac{1}{56}\left(35+9\,i\sqrt{7}\right)\text{Log}\left[4\,i+\left(i+\sqrt{7}\right)x+4\,i\,x^2\right]$$

Result (type 7, 101 leaves):

$$\frac{5\,\text{Log}[x]}{2}-\frac{1}{2}\text{RootSum}\Big[2+\#1+5\,\#1^2+\#1^3+2\,\#1^4\,\&,$$
$$\left(3\,\text{Log}[x-\#1]+19\,\text{Log}[x-\#1]\,\#1+\text{Log}[x-\#1]\,\#1^2+10\,\text{Log}[x-\#1]\,\#1^3\right)\Big/$$
$$\left(1+10\,\#1+3\,\#1^2+8\,\#1^3\right)\,\&\Big]$$

Problem 255: Result is not expressed in closed-form.

$$\int\frac{5+x+3\,x^2+2\,x^3}{x^2\left(2+x+5\,x^2+x^3+2\,x^4\right)}\,dx$$

Optimal (type 3, 281 leaves, 13 steps):

$$-\frac{35-9\,i\sqrt{7}}{28\,x}-\frac{35+9\,i\sqrt{7}}{28\,x}+\frac{11\left(9+5\,i\sqrt{7}\right)\text{ArcTanh}\left[\frac{i-\sqrt{7}+8\,i\,x}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right]}{4\sqrt{14\left(35-i\sqrt{7}\right)}}-$$
$$\frac{11\left(9-5\,i\sqrt{7}\right)\text{ArcTanh}\left[\frac{i+\sqrt{7}+8\,i\,x}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right]}{4\sqrt{14\left(35+i\sqrt{7}\right)}}-\frac{3}{56}\left(7-11\,i\sqrt{7}\right)\text{Log}[x]-\frac{3}{56}\left(7+11\,i\sqrt{7}\right)\text{Log}[x]+$$
$$\frac{3}{112}\left(7+11\,i\sqrt{7}\right)\text{Log}\left[4\,i+\left(i-\sqrt{7}\right)x+4\,i\,x^2\right]+\frac{3}{112}\left(7-11\,i\sqrt{7}\right)\text{Log}\left[4\,i+\left(i+\sqrt{7}\right)x+4\,i\,x^2\right]$$

Result (type 7, 109 leaves):

$$-\frac{5}{2\,x}-\frac{3\,\text{Log}[x]}{4}+\frac{1}{4}\text{RootSum}\Big[2+\#1+5\,\#1^2+\#1^3+2\,\#1^4\,\&,$$
$$\left(-35\,\text{Log}[x-\#1]+13\,\text{Log}[x-\#1]\,\#1-17\,\text{Log}[x-\#1]\,\#1^2+6\,\text{Log}[x-\#1]\,\#1^3\right)\Big/$$
$$\left(1+10\,\#1+3\,\#1^2+8\,\#1^3\right)\,\&\Big]$$

Problem 256: Result is not expressed in closed-form.

$$\int \frac{5+x+3 x^2+2 x^3}{x^3 \left(2+x+5 x^2+x^3+2 x^4\right)} \, dx$$

Optimal (type 3, 317 leaves, 13 steps):

$$-\frac{35-9 i \sqrt{7}}{56 x^2}-\frac{35+9 i \sqrt{7}}{56 x^2}+\frac{3 \left(7-11 i \sqrt{7}\right)}{56 x}+\frac{3 \left(7+11 i \sqrt{7}\right)}{56 x}+$$
$$\frac{\left(355-73 i \sqrt{7}\right) \text{ArcTanh}\left[\frac{i-\sqrt{7}+8 i x}{\sqrt{2 \left(35-i \sqrt{7}\right)}}\right]}{8 \sqrt{14 \left(35-i \sqrt{7}\right)}}-\frac{\left(355+73 i \sqrt{7}\right) \text{ArcTanh}\left[\frac{i+\sqrt{7}+8 i x}{\sqrt{2 \left(35+i \sqrt{7}\right)}}\right]}{8 \sqrt{14 \left(35+i \sqrt{7}\right)}}-$$
$$\frac{1}{16} \left(35-9 i \sqrt{7}\right) \text{Log}[x]-\frac{1}{16} \left(35+9 i \sqrt{7}\right) \text{Log}[x]+$$
$$\frac{1}{32} \left(35-9 i \sqrt{7}\right) \text{Log}\left[4 i+\left(i-\sqrt{7}\right) x+4 i x^2\right]+\frac{1}{32} \left(35+9 i \sqrt{7}\right) \text{Log}\left[4 i+\left(i+\sqrt{7}\right) x+4 i x^2\right]$$

Result (type 7, 116 leaves):

$$-\frac{5}{4 x^2}+\frac{3}{4 x}-\frac{35 \text{Log}[x]}{8}+$$
$$\frac{1}{8} \text{RootSum}\left[2+\#1+5 \#1^2+\#1^3+2 \#1^4 \&, \left(61 \text{Log}[x-\#1]+141 \text{Log}[x-\#1] \#1+\right.\right.$$
$$\left.\left.47 \text{Log}[x-\#1] \#1^2+70 \text{Log}[x-\#1] \#1^3\right) / \left(1+10 \#1+3 \#1^2+8 \#1^3\right) \&\right]$$

Problem 257: Result is not expressed in closed-form.

$$\int \frac{x^2 \left(3 a+b x^2\right)}{a^2+2 a b x^2+b^2 x^4+c^2 x^6} \, dx$$

Optimal (type 3, 19 leaves, 2 steps):

$$\frac{\text{ArcTan}\left[\frac{c x^3}{a+b x^2}\right]}{c}$$

Result (type 7, 87 leaves):

$$\frac{1}{2} \text{RootSum}\left[a^2+2 a b \#1^2+b^2 \#1^4+c^2 \#1^6 \&, \frac{3 a \text{Log}[x-\#1] \#1+b \text{Log}[x-\#1] \#1^3}{2 a b+2 b^2 \#1^2+3 c^2 \#1^4} \&\right]$$

Problem 387: Result is not expressed in closed-form.

$$\int \frac{x^2}{2-\left(1+x^2\right)^4} \, dx$$

Optimal (type 3, 157 leaves, 8 steps):

$$\frac{i\sqrt{1-i\,2^{1/4}}\ \text{ArcTan}\left[\frac{x}{\sqrt{1-i\,2^{1/4}}}\right]}{4\times 2^{3/4}}-\frac{i\sqrt{1+i\,2^{1/4}}\ \text{ArcTan}\left[\frac{x}{\sqrt{1+i\,2^{1/4}}}\right]}{4\times 2^{3/4}}-\frac{\sqrt{1+2^{1/4}}\ \text{ArcTan}\left[\frac{x}{\sqrt{1+2^{1/4}}}\right]}{4\times 2^{3/4}}+\frac{\sqrt{-1+2^{1/4}}\ \text{ArcTanh}\left[\frac{x}{\sqrt{-1+2^{1/4}}}\right]}{4\times 2^{3/4}}$$

Result (type 7, 61 leaves):

$$-\frac{1}{8}\text{RootSum}\left[-1+4\,\sharp1^2+6\,\sharp1^4+4\,\sharp1^6+\sharp1^8\,\&,\ \frac{\text{Log}[x-\sharp1]\,\sharp1}{1+3\,\sharp1^2+3\,\sharp1^4+\sharp1^6}\,\&\right]$$

Problem 388: Result is not expressed in closed-form.

$$\int\frac{x^2}{2-\left(1-x^2\right)^4}\,dx$$

Optimal (type 3, 157 leaves, 8 steps):

$$-\frac{\sqrt{-1+2^{1/4}}\ \text{ArcTan}\left[\frac{x}{\sqrt{-1+2^{1/4}}}\right]}{4\times 2^{3/4}}-\frac{i\sqrt{1-i\,2^{1/4}}\ \text{ArcTanh}\left[\frac{x}{\sqrt{1-i\,2^{1/4}}}\right]}{4\times 2^{3/4}}+\frac{i\sqrt{1+i\,2^{1/4}}\ \text{ArcTanh}\left[\frac{x}{\sqrt{1+i\,2^{1/4}}}\right]}{4\times 2^{3/4}}+\frac{\sqrt{1+2^{1/4}}\ \text{ArcTanh}\left[\frac{x}{\sqrt{1+2^{1/4}}}\right]}{4\times 2^{3/4}}$$

Result (type 7, 61 leaves):

$$-\frac{1}{8}\text{RootSum}\left[-1-4\,\sharp1^2+6\,\sharp1^4-4\,\sharp1^6+\sharp1^8\,\&,\ \frac{\text{Log}[x-\sharp1]\,\sharp1}{-1+3\,\sharp1^2-3\,\sharp1^4+\sharp1^6}\,\&\right]$$

Problem 389: Result is not expressed in closed-form.

$$\int\frac{x^2}{2+\left(1+x^2\right)^4}\,dx$$

Optimal (type 3, 188 leaves, 8 steps):

$$\frac{(-1)^{1/4}\sqrt{1-(-2)^{1/4}}\ \text{ArcTan}\left[\frac{x}{\sqrt{1-(-2)^{1/4}}}\right]}{4\times 2^{3/4}}-\frac{(-1)^{3/4}\sqrt{1+i\,(-2)^{1/4}}\ \text{ArcTan}\left[\frac{x}{\sqrt{1+i\,(-2)^{1/4}}}\right]}{4\times 2^{3/4}}-\frac{(-1)^{1/4}\sqrt{1+(-2)^{1/4}}\ \text{ArcTan}\left[\frac{x}{\sqrt{1+(-2)^{1/4}}}\right]}{4\times 2^{3/4}}+\frac{1}{8}\,i\left((-2)^{1/4}+\sqrt{2}\right)\sqrt{\frac{1+i}{(1+i)+2^{3/4}}}\ \text{ArcTan}\left[\sqrt{\frac{1+i}{(1+i)+2^{3/4}}}\,x\right]$$

Result (type 7, 61 leaves):

$$\frac{1}{8}\,\text{RootSum}\left[3+4\,\#1^2+6\,\#1^4+4\,\#1^6+\#1^8\,\&,\ \frac{\text{Log}\left[x-\#1\right]\,\#1}{1+3\,\#1^2+3\,\#1^4+\#1^6}\,\&\right]$$

Problem 390: Result is not expressed in closed-form.

$$\int \frac{x^2}{2+\left(1-x^2\right)^4}\,dx$$

Optimal (type 3, 188 leaves, 8 steps):

$$-\frac{(-1)^{1/4}\sqrt{1-(-2)^{1/4}}\,\text{ArcTanh}\left[\frac{x}{\sqrt{1-(-2)^{1/4}}}\right]}{4\times 2^{3/4}}+\frac{(-1)^{3/4}\sqrt{1+i\,(-2)^{1/4}}\,\text{ArcTanh}\left[\frac{x}{\sqrt{1+i\,(-2)^{1/4}}}\right]}{4\times 2^{3/4}}+\frac{(-1)^{1/4}\sqrt{1+(-2)^{1/4}}\,\text{ArcTanh}\left[\frac{x}{\sqrt{1+(-2)^{1/4}}}\right]}{4\times 2^{3/4}}-\frac{1}{8}\,i\left((-2)^{1/4}+\sqrt{2}\right)\sqrt{\frac{1+i}{(1+i)+2^{3/4}}}\,\text{ArcTanh}\left[\sqrt{\frac{1+i}{(1+i)+2^{3/4}}}\,x\right]$$

Result (type 7, 61 leaves):

$$\frac{1}{8}\,\text{RootSum}\left[3-4\,\#1^2+6\,\#1^4-4\,\#1^6+\#1^8\,\&,\ \frac{\text{Log}\left[x-\#1\right]\,\#1}{-1+3\,\#1^2-3\,\#1^4+\#1^6}\,\&\right]$$

Problem 391: Result is not expressed in closed-form.

$$\int \frac{1-x^2}{a+b\left(1-x^2\right)^4}\,dx$$

Optimal (type 3, 663 leaves, 16 steps):

$$-\frac{\text{ArcTan}\left[\frac{b^{1/8}x}{\sqrt{(-a)^{1/4}-b^{1/4}}}\right]}{4\sqrt{-a}\sqrt{(-a)^{1/4}-b^{1/4}}\;b^{3/8}}-\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-b^{1/4}}\;\text{ArcTan}\left[\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+b^{1/4}}-\sqrt{2}\,b^{1/8}x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-b^{1/4}}}\right]}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}\;b^{3/8}}+$$

$$\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-b^{1/4}}\;\text{ArcTan}\left[\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+b^{1/4}}+\sqrt{2}\,b^{1/8}x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-b^{1/4}}}\right]}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}\;b^{3/8}}+\frac{\text{ArcTanh}\left[\frac{b^{1/8}x}{\sqrt{(-a)^{1/4}+b^{1/4}}}\right]}{4\sqrt{-a}\sqrt{(-a)^{1/4}+b^{1/4}}\;b^{3/8}}+$$

$$\left(\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+b^{1/4}}\;\text{Log}\left[\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+b^{1/4}}\;b^{1/8}x+b^{1/4}x^2\right]\right)\Big/$$

$$\left(8\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}\;b^{3/8}\right)-$$

$$\left(\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+b^{1/4}}\;\text{Log}\left[\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+b^{1/4}}\;b^{1/8}x+b^{1/4}x^2\right]\right)\Big/$$

$$\left(8\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}\;b^{3/8}\right)$$

Result (type 7, 63 leaves):

$$-\frac{\text{RootSum}\left[a+b-4b\,\#1^2+6b\,\#1^4-4b\,\#1^6+b\,\#1^8\,\&,\ \frac{\text{Log}[x-\#1]}{\#1-2\,\#1^3+\#1^5}\,\&\right]}{8b}$$

Problem 392: Result is not expressed in closed-form.

$$\int\frac{1-x^2}{a+b\left(-1+x^2\right)^4}\,dx$$

Optimal (type 3, 663 leaves, 17 steps):

$$-\frac{\text{ArcTan}\left[\frac{b^{1/8}\,x}{\sqrt{(-a)^{1/4}-b^{1/4}}}\right]}{4\sqrt{-a}\,\sqrt{(-a)^{1/4}-b^{1/4}}\;b^{3/8}}-\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-b^{1/4}}\;\text{ArcTan}\left[\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+b^{1/4}}-\sqrt{2}\,b^{1/8}\,x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-b^{1/4}}}\right]}{4\sqrt{2}\,\sqrt{-a}\,\sqrt{\sqrt{-a}+\sqrt{b}}\;b^{3/8}}+$$

$$\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-b^{1/4}}\;\text{ArcTan}\left[\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+b^{1/4}}+\sqrt{2}\,b^{1/8}\,x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-b^{1/4}}}\right]}{4\sqrt{2}\,\sqrt{-a}\,\sqrt{\sqrt{-a}+\sqrt{b}}\;b^{3/8}}+\frac{\text{ArcTanh}\left[\frac{b^{1/8}\,x}{\sqrt{(-a)^{1/4}+b^{1/4}}}\right]}{4\sqrt{-a}\,\sqrt{(-a)^{1/4}+b^{1/4}}\;b^{3/8}}+$$

$$\left(\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+b^{1/4}}\;\text{Log}\left[\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt{2}\,\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+b^{1/4}}\;b^{1/8}\,x+b^{1/4}\,x^2\right]\right)\Big/$$

$$\left(8\sqrt{2}\,\sqrt{-a}\,\sqrt{\sqrt{-a}+\sqrt{b}}\;b^{3/8}\right)-$$

$$\left(\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+b^{1/4}}\;\text{Log}\left[\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt{2}\,\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+b^{1/4}}\;b^{1/8}\,x+b^{1/4}\,x^2\right]\right)\Big/$$

$$\left(8\sqrt{2}\,\sqrt{-a}\,\sqrt{\sqrt{-a}+\sqrt{b}}\;b^{3/8}\right)$$

Result (type 7, 63 leaves):

$$-\frac{\text{RootSum}\left[a+b-4\,b\,\#1^2+6\,b\,\#1^4-4\,b\,\#1^6+b\,\#1^8\,\&,\ \frac{\text{Log}[x-\#1]}{\#1-2\,\#1^3+\#1^5}\,\&\right]}{8\,b}$$

Problem 393: Result is not expressed in closed-form.

$$\int\frac{\left(1+x^2\right)^2}{a\,x^6+b\,\left(1+x^2\right)^3}\,dx$$

Optimal (type 3, 168 leaves, ? steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{a^{1/3}+b^{1/3}}\;x}{b^{1/6}}\right]}{3\sqrt{a^{1/3}+b^{1/3}}\;b^{5/6}}+\frac{\text{ArcTan}\left[\frac{\sqrt{-(-1)^{1/3}\,a^{1/3}+b^{1/3}}\;x}{b^{1/6}}\right]}{3\sqrt{-(-1)^{1/3}\,a^{1/3}+b^{1/3}}\;b^{5/6}}+\frac{\text{ArcTan}\left[\frac{\sqrt{(-1)^{2/3}\,a^{1/3}+b^{1/3}}\;x}{b^{1/6}}\right]}{3\sqrt{(-1)^{2/3}\,a^{1/3}+b^{1/3}}\;b^{5/6}}$$

Result (type 7, 95 leaves):

$$\frac{1}{6}\,\text{RootSum}\left[b+3\,b\,\#1^2+3\,b\,\#1^4+a\,\#1^6+b\,\#1^6\,\&,\ \frac{\text{Log}[x-\#1]+2\,\text{Log}[x-\#1]\,\#1^2+\text{Log}[x-\#1]\,\#1^4}{b\,\#1+2\,b\,\#1^3+a\,\#1^5+b\,\#1^5}\,\&\right]$$

Problem 421: Result more than twice size of optimal antiderivative.

$$\int\frac{2}{-1+4\,x^2}\,dx$$

Optimal (type 3, 6 leaves, 2 steps):

$$-\text{ArcTanh}[2x]$$

Result (type 3, 23 leaves):

$$2\left(\frac{1}{4}\text{Log}[1-2x]-\frac{1}{4}\text{Log}[1+2x]\right)$$

Problem 491: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^2}{1+\left(-1+x^2\right)^2}\,dx$$

Optimal (type 3, 188 leaves, 10 steps):

$$-\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\ \text{ArcTan}\left[\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2x}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right]+\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\ \text{ArcTan}\left[\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2x}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right]+$$

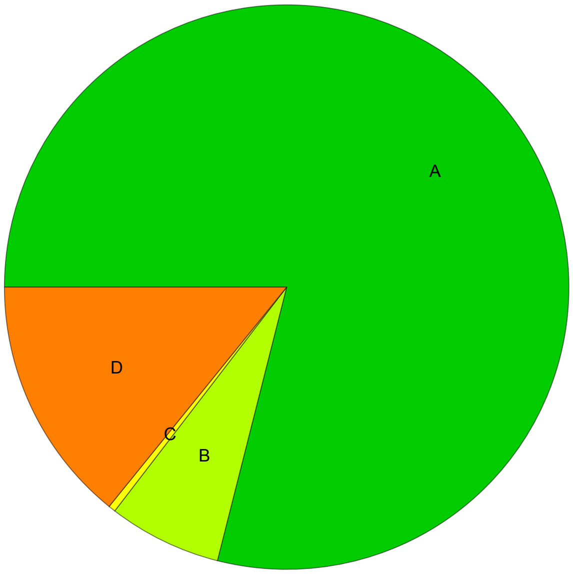
$$\frac{\text{Log}\left[\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}\ x+x^2\right]}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\text{Log}\left[\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)}\ x+x^2\right]}{4\sqrt{2\left(1+\sqrt{2}\right)}}$$

Result (type 3, 39 leaves):

$$-\frac{\text{ArcTan}\left[\frac{x}{\sqrt{-1-i}}\right]}{(-1-i)^{3/2}}-\frac{\text{ArcTan}\left[\frac{x}{\sqrt{-1+i}}\right]}{(-1+i)^{3/2}}$$

Summary of Integration Test Results

494 integration problems



A - 390 optimal antiderivatives

B - 32 more than twice size of optimal antiderivatives

C - 2 unnecessarily complex antiderivatives

D - 70 unable to integrate problems

E - 0 integration timeouts